SESSION PLAN OF CC 1

Session – 1 (2023 - 2024) (odd Semester)

Session Name : General Equation Of the Second Degree

Teacher Name : Mr. Snehendu Mandal

Department : Mathematics

Subject / Course Name : Analytical Geometry of 2D & 3D

Course Code : CC 1

Level Of Students : B.Sc. Math (Hons) , 1st Sem

Contact No : < 8436475960

Session Objectives :

At the end of the session students will be able to :

- ✓ *Classify the Conics*
- ✓ *Transformed to its Canonical Forms to know the Conics easily*
- ✓ *The invaariants under the orthogonal transformation*
- ✓ *Find The Nature of the conics*
- ✓ *Know about Degenerate & non – degenerate conics*
- ✓ *Understand about Central Conics & non- Central Conics*
- ✓ *Understand the Centre of the conics*
- ✓ *Determine Length & position of the axes of a central conic*

Teaching –Learning Material

- ➢ *White Board & Marker Pen*
- ➢ *Brainstorming*
- ➢ *Referrence Book*
- ➢ *Study Notes*
- ➢ *Reference Notes*

Session Plan

Time (in min)	Content	Learning aid & Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardners)
10	Review of previous lession	Brainstorming	Question Answer	Listens Participate	Remembering Understanding
10	Idea about 2^{nd} deg homogeneous equ , discriminant Δ & D , invariance under orthogonal transformation	Brainstorming Demonstration Group Discussion	Expains Demonstarte	Listens Discuss	Understanding Remembering
10	Nature of Conics Canonical forms	Case -Study Demonstration	Expains & Facilitates	Analyze Understand	Analyzing Understanding Applying
08	Central & Non – Central Conics	Brainstorming Demonstration	Deductions & Explains	Listens Participates Discuss	Remembering Visual – spatial Intrapersonal
07	Centre of a Conic Degeneracy Non – degeneracy	Case -Study Group discussion	Demonstrate Analyze	Formulate Analyze	Understanding Intrapersonal
15	Various Problems on nature of Conics & Canonical forms	Innovative Conclusion	Hints for Solution	Participates Group - Discuss	Conceptal Approach Remembering Applying knowledge to solve problems

Topic : General Equ of 2^{nd} Degree   Teacher Name : Snehendu Mandal
College name : Shyampur Siddheswari Mahavidyalaya , Ajodhya , Howrah

Session Inputs

Consider the General equation of 2^{nd} degree

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

We shall show this will represent a conic. Let us first transform the equation by rotation of axes such that we can get rid of the term xy . For this we , rotate the co ordinate axes through an angle θ , then we have

$$\left.\begin{aligned} x &= x'cos\theta - y'sin\theta \\ y &= y'cos\theta + x'sin\theta \end{aligned}\right\}$$

the new co ordinate axes being (x' , y')

now applying the transformation equ 1 becomes

$$a\,(x'cos\theta - y'sin\theta)^2 + b\,(y'cos\theta + x'sin\theta)^2 + 2h\,(x'cos\theta - y'sin\theta)(y'cos\theta + x'sin\theta)$$

$$+ 2g\,(x'cos\theta - y'sin\theta) + 2f\,(y'cos\theta + x'sin\theta) + c = 0 \quad \ldots\ldots\ldots\ldots \quad (2)$$

Let us choose θ such that the coefficient of the term containing $x'y'$ vanish in (2)

$$2h cos2\theta - (a-b)sin2\theta = 0$$

$$tan\ 2\theta = \frac{2h}{a-b} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

then (2) reduces to the form

$$a'x'^2 + b'y'^2 + 2g'x' + 2f'y' + c' = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

SMandal

Suggested Activity

Case – I *let* $a' \neq 0$, $b' \neq 0$

Then the equ (4) can be put as

$$a'\left(x' + \frac{g'}{a'}\right)^2 + b'\left(y' + \frac{f'}{b'}\right)^2 = \frac{g'^2}{a'} + \frac{f'^2}{b'} - c' = k \ (const)$$

Now shifting the origin to $\left(\frac{g'}{a'}, -\frac{f'}{b'}\right)$ *without changing the direction of axes this equ becomes*

$$a'X^2 + b'Y^2 = k \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (5)$$

a) *if* $k = 0$ (5) *represents two straight lines Real or Imaginary imaginary lines also known as point ellipse*

b) *If* $k \neq 0$ *then* (5) *becomes*

$$\frac{X^2}{\frac{k}{a'}} + \frac{y^2}{\frac{k}{b'}} = 1$$

i) *If both* $\frac{k}{a'}$ & $\frac{k}{b'}$ *are positive then* (5) *represents an ellipse .*

In addition if $a' = b'$ *then this will be a circle .*

ii) *if one of* $\frac{k}{a'}$ & $\frac{k}{b'}$ *be positive , other be negative , then this will represents a hyperbola*

In addition if $a' + b' = 0$ *it will be a Rectangular Hyperbola .*

iii) If both $\frac{k}{a'}$ & $\frac{k}{b'}$ *be negative then (5) becomes imaginary ellipse .*

Case – 2 : *if one of* a' & b' *be zero say* $a' = 0$

(4) can be written as $b'(y' + \frac{f'}{b'})^2 = -2g'x' + \frac{f'^2 - b'c'}{b'}$ ………………… *(6)*

1) If $g' = 0$ *then (6) represents a pair of parallel straight lines*

, provided $\frac{f'^2 - b'c'}{b'} \neq 0$

ii) If $g' = 0$ & $\frac{f'^2 - b'c'}{b'}$ *is negative then it has no geometrical locus*

iii) If $g' = 0$ & $\frac{f'^2 - b'c'}{} = 0$ *then it wil represents a coincident lines .*

iv) if $g' \neq 0$ *then (6) becomes*

$$(y' + \frac{f'}{b'})^2 = -2\frac{g'}{b'}(x' - \frac{f'^2 - b'c'}{2b'g'})$$

Shifting the origin to $(\frac{f'^2 - b'c'}{2b'g'} , -\frac{f'}{b'})$

Equation becomes $Y^2 = -2\frac{g'}{b'}X$

This represents a parabola .

Simple problems on general equation of 2nd degree

a) ***Find the nature of the following conics :***

1) $3x^2 + 2y^2 - 5xy + 5x - 8y + 10 = 0$

ii) $6x^2 + 5y^2 - 25xy + 10x - 18y + 20 = 0$

iii) $-5x^2 + 12y^2 - 12xy + 8x - 12y + 24 = 0$

b) ***Find the canonical forms of the followings :***

1) $3x^2 + 2y^2 - 15xy + 5x - 8y + 100 = 0$

ii) $-16x^2 + 5y^2 - 25xy + 10x - 18y + 26 = 0$

iii) $25x^2 + 12y^2 - 20xy + 8x - 12y + 40 = 0$

c) ***Find the values of b & g such that the equation***

$$4x^2 + 8xy + by^2 + 2gx + 4y + 1 = 0$$

Represents i) a conic without any centre ii) a conic having infinitely many centres .

Topic : General Equ of 2nd Degree Teacher Name : Snehendu Mandal
College name : Shyampur Siddheswari Mahavidyalaya , Ajodhya , Howrah

References :

1) *Advanced analytical Geometry , J.G . Chakraborty , & P. R . Ghosh*
2) *Advanced Geometry & Vector Analysis , Ghosh & Chakroborty*
3) *Analytical Geometry of 2D & 3D & Vector Calculus by R. M . Khan*
4) *Advanced Analytical Geometry by S . L . Loney*

Session – 31

Session Name: Mapping or Function

Author Name: Mrs. Rini Maiti

Department: Mathematics

Subject/Course: Algebra

Course Code: CC-2

Level of students: B.Sc. Mathematics Hons 2nd Semester

Cell Number: < 8016551103>

Session Objectives :

At the end of this session, the learner will be able to:

- Recall the definition of domain, co-domain and range of a function or of a mapping
- Find the domain, co-domain and range of a function or of a mapping
- Explain why any function or a mapping is injective/surjective/bijective
- Check whether a mapping is into/onto

Teaching-Learning Material:

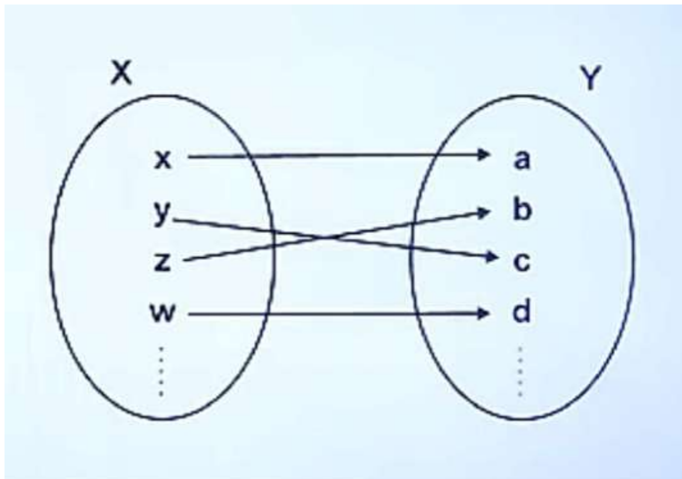
- White board and Marker
- Brainstorming
- Hand-Written Notes
- Reference Books
- Analyzing

Session Plan :

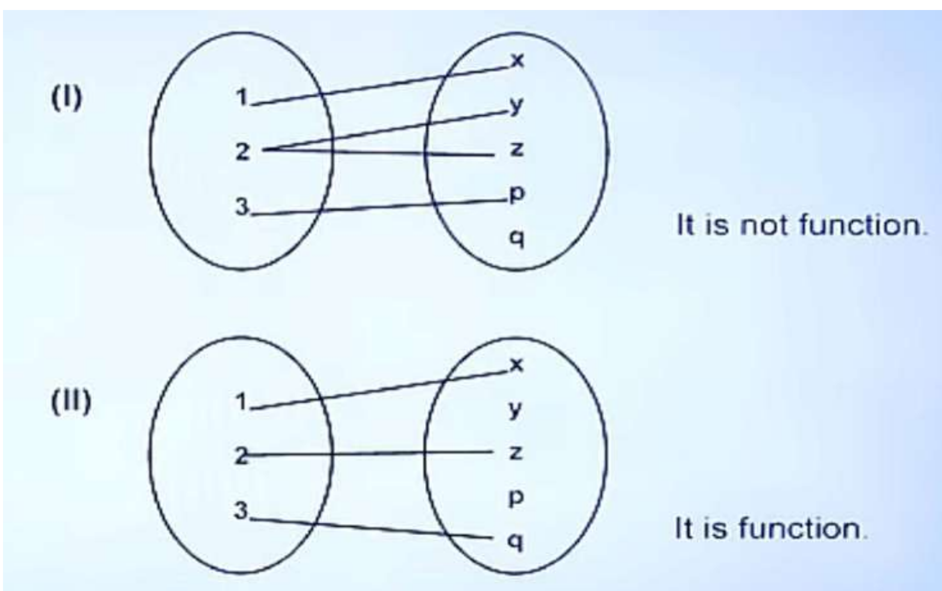
Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Idea of a mapping or a function	Discussions with figure drawing	Explains	Listens Participates Discusses	Understanding Interpersonal Intrapersonal Verbal-linguistic
10	Concept of domain, co-domain and range of a function or of a mapping	Discussions by giving various eamples	Explains	Listens Participates Discusses	Remembering Understanding Analyzing Interpersonal Intrapersonal Verbal-linguistic
10	Concept of different kind of function: into mapping & onto mapping	Discussions by giving various examples. Case - study Group-Discussion	Facilitates Explains	Listens, Understands, Analyzes, Discusses, Participates	Applying Analyzing
15	Concept of different kind of function: injective mapping, surjective mapping, bijective mapping	Demonstration Discussions	Explains	Listens, Understands, Discusses	Remembering Understanding
15	Application of various method for checking injective mapping, surjective mapping	Case - study Group-Discussion	Monitors Facilitates	Solve problems Participates	Understanding Applying Analyzing

Session Inputs :

Function or Mapping: Let X & Y be two non-empty sets. A mapping f from X to Y is a rule that assigns each element x of X to a definite element y of Y. It is written as $f : X \to Y$.



Example: Let A = {1, 2, 3} & B ={x, y, z, p, q}

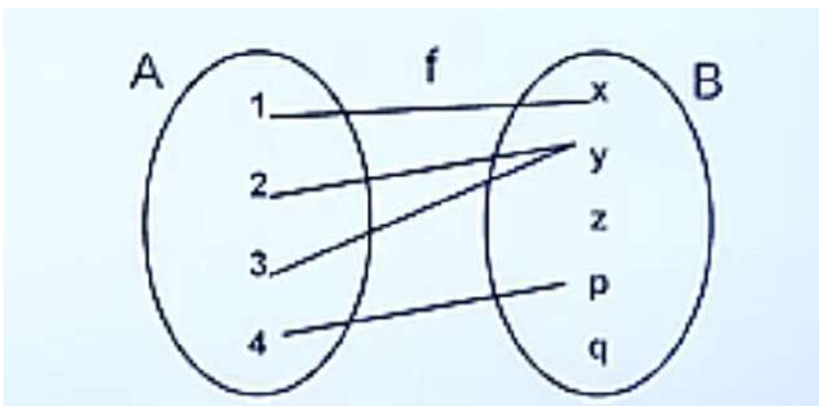


Domain & Co-domain of function: If f is function from A to B, then A is called domain and B is called co-domain.

Range of function: If f is function from A to B, then the set of all those elements of B which are related with the elements of A is called the range of f & it is denoted by f(A).

Evidently, Range(f) $\subset$ B.

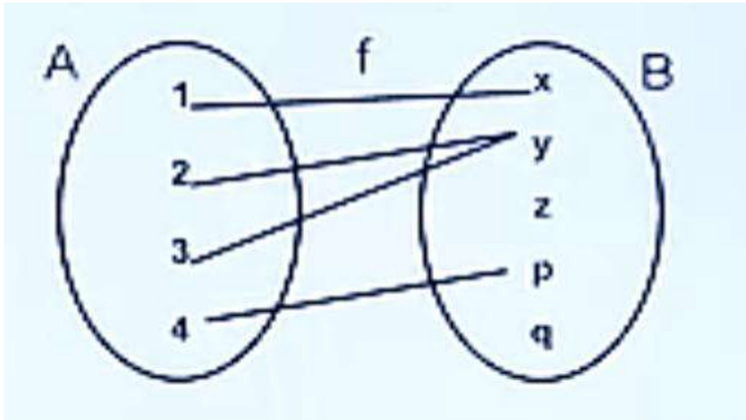
Example: Let A = {1, 2, 3, 4} & B = {x, y, z, p, q}.



Here domain of f = {1, 2, 3, 4}, co- domain of f = {x, y, z, p, q } & range of f = {x, y, p}.

Into Mapping: A mapping f : A→B is said to be an into mapping if f(A) is a proper subset of B. In this case we say that f maps A into B.

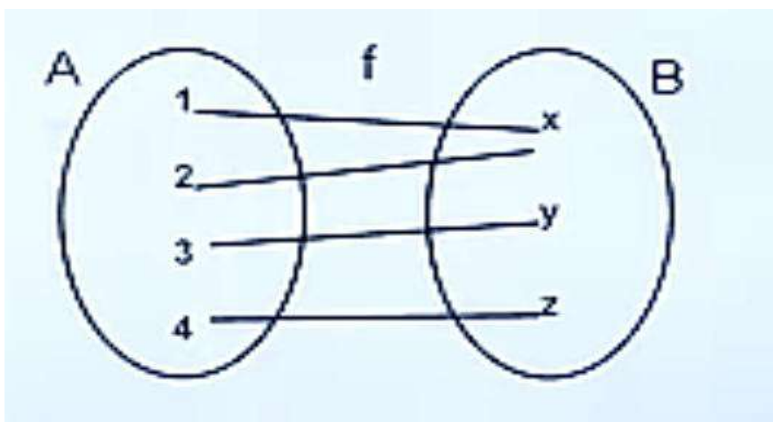
Example: Let A = {1, 2, 3, 4} & B = $\{x, y, z, p, q\}$.



Here f(A) = {x, y, z} is a proper subset of B. So it is an into mapping.

Onto Mapping: A mapping f : A→B is said to be an onto mapping if f(A) = B. In this case we say that f maps A onto B.

Example: Let A = {1, 2, 3, 4} & B = $\{x, y, z\}$.



Here f(A) = {x, y, z}= B. So it is an onto mapping.

Injective or one-to-one mapping: A mapping f : A→B is said to be an injective or one-to-one mapping if for each pair of distinct elements of A, their f-images are distinct.

In this case, each element of B has at most one pre-image.

Surjective or onto mapping: A mapping f : A→B is said to be surjective or onto mapping if f(A) = B.

In this case, each element of B has at least one pre-image.

Bijective mapping: A mapping f : A→B is said to be bijective if f is both injective and surjective.

In this case, each element of B has exactly one pre-image.

Suggested Activity:

Students are involving to understand different kind of mappings.

They are trying to define domain of a function. They are trying to check whether a mapping is injective/surjective/bijective.

Simple Problems on Mapping:

Problem1: Find the domain of the function

(i) $f(x) = \frac{x}{1+x^2}$ (ii) $f(x) = \sqrt{x-4}$

Problem 2: Find the range of the function

(i) $f(x) = \frac{x}{1-x}$ (ii) $f(x) = \sqrt{9-x^2}$

Problem 3: Let f: R→R be defined by $f(x) = 3x + 1, x \in$R. Examine if f is

(i) injective (ii) surjective.

Problem 4: Give an example of an infinite set S and a mapping f: S→S such that

(i) f is injective but not surjective

(ii) f is surjective but not injective.

Summary:

In this session, we learnt

- How to find the domain, co-domain and range of a function or of a mapping
- To examine whether a mapping is into/onto
- To explain with proper justification of any mapping being injective/ surjective/bijective

Assignment:

- Let $f = \left\{(x, y) \in R \times R : y = \frac{1}{x}\right\}$. Let us examine if f is a mapping from $R \ to \ R$.
- Let f : Z→Z be defined by f(x)=2x, x∈Z. Show that f is an into mapping.
- Let f : Z→Z be defined by f(x)=|x|, x∈Z. Show that f is an into mapping.
- Let f : Z→Z be defined by f(x)=x+1, x∈Z. Show that f is an onto mapping.
- Let f : R→R be defined by f(x)=sinx, x∈R. Show that the mapping f is neither injective nor surjective.
- Let f : R→Z be defined by f(x)=[x], x∈R. Show that the mapping f is surjective but not injective.

References:

- Higher Algebra, Abstract and Linear – S.K.Mapa
- Algrbra- R.M.Khan
- A Course on Abstract Algebra – Vijay K Khanna & S.K.Bhambri
- Abstract Algebra – L.N.Herstein
- Higher Algebra – S.Bernard & J.M.Clild

Session – 19

Session Name: **Real Sequence**

Author Name: **Mrs. Rini Maiti**

Department: **Mathematics**

Subject/Course: **Real Analysis**

Course Code: **CC-3**

Level of students: **B.Sc. Mathematics Hons 3^{nd} Semester**

Cell Number: **< 8016551103>**

Session Objectives :

At the end of this session, the learner will be able to:

- Find the range of any real sequence
- Recall the definition of a bounded above, bounded below and bounded sequence
- Explain when a sequence will be unbounded above, unbounded below
- Define convergent sequence & divergent sequence
- Evaluate limit of a sequence when it exists

Teaching-Learning Material:

- White board and Marker
- Brainstorming
- Hand-written Notes
- Reference Books
- Analyzing

Session Plan :

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
15	Idea of sequence of real numbers, range of a sequence	Discussions with figure drawing & by giving various examples	Explains	Listens Participates Discusses	Understanding Interpersonal Intrapersonal Verbal-linguistic
15	Concept of bounded above, bounded below, unbounded above, unbounded below sequence	Discussions by giving various examples	Explains	Listens Participates Discusses	Remembering Understanding Analyzing Interpersonal Intrapersonal Verbal-linguistic
20	Concept of limit of a sequence & uniqueness of limit of a sequence	Demonstration Discussions by giving various examples Group-Discussion	Facilitates Explains	Listens, Understands, Analyzes, Discusses, Participates	Applying Analyzing
10	Concept of convergent, divergent & constant sequence	Demonstration Discussions by giving various examples	Explains	Listens, Understands, Discusses	Remembering Understanding Analyzing

Session Inputs :

Sequence of real numbers: A sequence of real numbers is a function whose domain is the set N of natural numbers and co- domain is the set R of real numbers. That is, a mapping f : N→R which is defined by f(n) = a_n for all n∈N is a sequence of real numbers.

A sequence is generally denoted by {f(n)}={ a_n}, where { a_n}= {a_1, a_2 , …. , a_n , …….}.

f(1)= a_1 f(2)= a_2 f(3)= a_3

Range of a sequence: The set of all distinct terms of a sequence is called its range.

The range of the real sequence {f(n)} is a subset of the set R of real numbers, denoted by {f(n) : n∈N }.

Note: Range of a sequence may or may not be finite but it is always countable.

Example: (i) Let f : N→R be defined by f(n)= $(-1)^n$, n∈N. The sequence is $\{(-1)^n\}$. It is also denoted by $\{-1, 1, -1, 1, -1, 1, \ldots\ldots..\}$. The range of this sequence is $\{-1, 1\}$.

(ii) Let f : N→R be defined by f(n)=n, n∈N. The sequence is {n}. It is also denoted by $\{1, 2, 3, 4, 5, \ldots\ldots..\}$. The range of this sequence is $\{1, 2, 3, 4, 5, \ldots\ldots\}$.

(iii) Let f : N→R be defined by f(n)= $\sin\frac{n\pi}{2}$, n∈N. The sequence is $\{1, 0, -1, 0, 1, 0, \ldots\ldots\ldots\}$. The range of this sequence is $\{-1, 0, 1\}$.

Bounded Above Sequence: A real sequence {f(n)}is said to be bounded above if there exists a real number G such that f(n)≤G for all n∈N. G is said to be an upper bound of the sequence.

The least upper bound of the sequence {f(n)}is said to be the supremum of the sequence{f(n)} and it is denoted by sup{f(n)}.

Bounded Below Sequence: A real sequence {f(n)}is said to be bounded below if there exists a real number g such that f(n)≥g for all n∈N. g is said to be an lower bound of the sequence.

The greatest lower bound of the sequence {f(n)}is said to be the infimum of the sequence{f(n)} and it is denoted by inf{f(n)}.

Bounded Sequence: A real sequence {f(n)}is said to be a bounded sequence if there exists real numbers G and g such that g≤f(n)≤G for all n∈N.

Unbounded Above Sequence: A real sequence {f(n)}which is not bounded above is said to be unbounded above sequence.

Unbounded Below Sequence: A real sequence {f(n)}which is not bounded below is said to be unbounded below sequence.

Example: (i) The sequence $\{\frac{1}{n}\}$is a bounded sequence. 0 is the greatest lower bound and 1 is the least upper bound of the sequence.

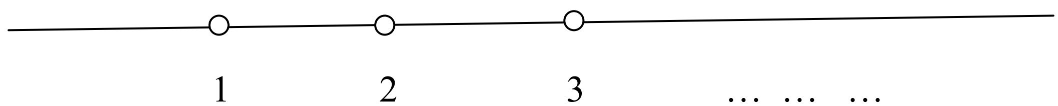
(ii) The sequence $\{n^2\}$is bounded below and unbounded above. Here sup{f(n)}$=\infty$ and inf{f(n)}$=1$.

(iii) The sequence $\{-2n\}$is bounded above and unbounded below. Here sup{f(n)}$=-2$ and inf{f(n)}$=-\infty$.

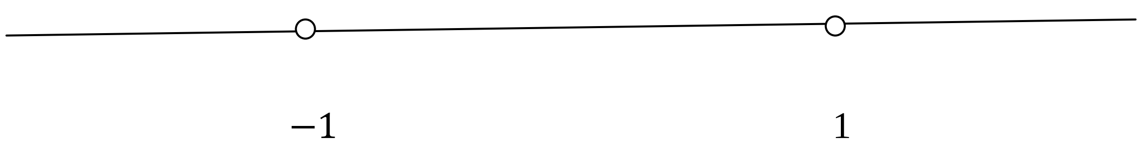
(iv) The sequence $\{(-1)^n n\}$is unbounded above and unbounded below. Here sup{f(n)}$=\infty$ and inf{f(n)}$=-\infty$.

Note: All terms of a sequence can be put on real line.

Example: (i) Let $\{ a_n\}= \{n\}$



(ii) Let $\{ a_n\}= \{(-1)^n\}$.



Limit of a sequence: Let {f(n)}be a real sequence. A real number l is said to be a limit of the sequence {f(n)}if corresponding to a pre-assigned positive ϵ there exists a natural number k(depending on ϵ) such that

$$| \mathrm{f(n)} - l| < \epsilon \ for\ all\ n \geq k$$

i.e. $l - \epsilon < \mathrm{f(n)} < l + \epsilon \ for\ all\ n \geq k.$

Example: (i) Limit of the sequence $\{ \frac{1}{n} \}$is 0.

(ii) Limit of the sequence $\{(-1)^n\}$ does not exist.

Note: A sequence can have at most one limit.

Convergent Sequence: A real sequence $\{ f(n)\}$is said to be a convergent sequence if it has limit $l \in R$.

And we write $\lim_{n\to\infty} f(n) = l$.

Example: The sequence $\{ \frac{1}{n} \}$is convergent.

Divergent Sequence: A real sequence $\{ f(n)\}$is said to be a divergent sequence if it is not convergent.

Example: The sequence $\{ n^2\}$is divergent.

Constant Sequence: A real sequence $\{ f(n)\}$is said to be a constant sequence if f(n)=c for all n∈N.

Example: The sequence { 2 }is a constant sequence.

Suggested Activity:

Students are involving to understand different types sequences.

They are trying to calculate the limit of a sequence if it exists. They are trying to check whether a sequence is convergent/ divergent/constant.

Simple Problems on Mapping:

Problem1: Find the range of the sequences

(i) $f(n) = \frac{n}{n+1}$ (ii) $f(n) = \frac{(-1)^n}{n}$

Problem 2: Find the limit of the sequences

(i) $f(n) = \frac{n^2+1}{n^2}$ (ii) $f(n) = 5$

Problem 3: Give an example of a sequence which is bounded but not convergent.

Summary:

In this session, we learnt

- How to calculate the range of a sequence
- To examine whether a sequence is convergent/ divergent/constant
- To explain with proper justification why a sequence is bounded / bounded above / bounded below / unbounded above / unbounded below.

Assignment:

- Let f : N→R be defined by $f(n) = \frac{n^2+1}{n^2}$, $n \in$ R. Find the range of the sequence $\{ f(n)\}$.
- Let f : N→R be defined by $f(n) = \frac{(-1)^n}{n}$, $n \in$ R. Find the limit of the sequence $\{ f(n)\}$.
- Let f : N→R be defined by $f(n) = \frac{3n^2+2n+1}{n^2+1}$, $n \in$ R. Prove that the limit of the sequences $\{ f(n)\}$ is 3.
- Give an example of a sequence which is bounded above but unbounded below.
- Give an example of a sequence which is unbounded above but bounded below.

References:

- Introduction to Real Analysis – S.K.Mapa
- Mathematical Analysis – S.C.Malik & Savita Arora
- Introduction to Real Analysis – Bartle & Sherbert
- Real Analysis – N.P.Bali
- Principles of Mathematical Analysis – Walter Rudin

SESSION PLAN OF CC4

Session – 1 (2022-2023) (Even Semester)

Session Name : Permutations

Teacher Name : Mr. Snehendu Mandal

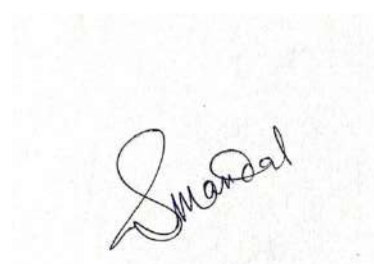
Department : Mathematics

Subject / Course : Group Theory -I

Course Code : CC 4

Level Of Students : B.Sc. Math (Hons) , 2nd Sem

Contact No :< 8436475960 >



Session Objectives :

At the end of the session student will be able to

- *Know the Basics about Permutations*
- *Express a permutation in a cycle notations*
- *Know that a permutation is nothing but a Bijective mapping on a set S*
- *Understand multiplication of permutations & inverse of a permutation*
- *Express a permutation as a product of disjoint cycles*
- *Understand about transposition or a 2 – cycle*
- *Integral powers of permutations*
- *Even & Odd permutations*
- *Order of a permutation*
- *The identity & the inverse of a permutation Group*

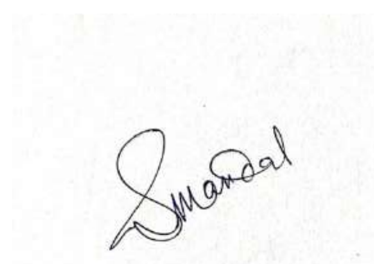
Teaching –Learning Material

- *Black Board & Chalk*
- *Brainstorming*
- *Referrence Book*
- *Notes*
- *Pen & Paper*
- *White Board & Marker*

Session Plan

Time (in min)	Content	Learning aid & Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardners)
05	Review of previous lession	Brainstorming	Explain Question Answer	Listens writting Participate	Remembering Understanding
10	Idea about Bijective mapping & then know the basics of permutation	Brainstorming Deductions & Explains	Expains Demonstarte Example Problem solve	Listens Discuss Analyze	Understanding Analyzing Applying Participates
10	Know Multipliation , Inverse , Cycle of permutation	Case -Study Formulate Analyze	Expains & Facilitates	Analyze Understand	Analyzing Understanding Applying
10	Integral power , Product of disjoint cycles , Transposition	Brainstorming Demonstration	Deductions & Explains	Listens Participates Discuss	Remembering Visual – spatial Intrapersonal
10	Even , Odd & order of permutation	Case -Study Group discussion	Demonstrate Analyze	Formulate Analyze	Understanding Intrapersonal
15	Various Problems on Permutations	Innovative Conclusion	Hints for Solution	Participates Group - Discuss	Conceptal Approach Remembering Applying knowledge to solve problems

Topic : Permutations TeacherName : Snehendu Mandal , College name : Shyampur Siddheswari Mahavidyalaya , Ajodhya , Howrah



Session Inputs :

Definition : *Let s be a non- empty set . A Bijective mapping $f : S \rightarrow S$ is said to be a Permutation on s*

Let $S = \{ a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_{n-1}, a_n \}$. Then the number of bijections from S onto S is n ! . Let one Such Bijection be f that maps a_i to $f(a_i)$, this permutation f is denoted by the symbol

$$\begin{pmatrix} a_1 & a_2 & a_3 & \ldots\ldots\ldots\ldots & a_n \\ f(a_1) & f(a_2) & f(a_3) & \ldots\ldots\ldots\ldots & f(a_n) \end{pmatrix}$$

Let $S = \{1, 2, 3, 4\}$

Let $\pi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ $\qquad \pi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

Multiplication of Permutations *:*

Let $f = \begin{pmatrix} a_1 & a_2 & \ldots\ldots\ldots\ldots & a_n \\ f(a_1) & f(a_2) & \ldots\ldots\ldots\ldots & f(a_n) \end{pmatrix}$

$g = \begin{pmatrix} a_1 & a_2 & \ldots\ldots\ldots\ldots & a_n \\ g(a_1) & g(a_2) & \ldots\ldots\ldots\ldots & g(a_n) \end{pmatrix}$

Then $f.g = \begin{pmatrix} a_1 & a_2 & \ldots\ldots\ldots\ldots & a_n \\ f(g(a_1)) & f(g(a_2)) & \ldots\ldots\ldots\ldots & f(g(a_n)) \end{pmatrix}$

$g.f = \begin{pmatrix} a_1 & a_2 & \ldots\ldots\ldots\ldots & a_n \\ g(f(a_1)) & g(f(a_2)) & \ldots\ldots\ldots\ldots & g(f(a_n)) \end{pmatrix}$

Inverse of a Permutation :

Let $f : S \to S$ be a permutation , then definitely its inverse exists as it is a bijection

Let inverse be $f^{-1} : S \to S$ such that $f.f^{-1} = i$

If by f , $a_i \to a_j$ then by f^{-1} , $a_j \to a_i$

That is if

$$g = \begin{pmatrix} a_1 & a_2 & \ldots\ldots\ldots\ldots & a_n \\ g(a_1) & g(a_2) & \ldots\ldots\ldots\ldots & g(a_n) \end{pmatrix}$$

Then $g^{-1} = \begin{pmatrix} g(a_1) & g(a_2) & \ldots\ldots\ldots\ldots & g(a_n) \\ (a_1) & (a_2) & \ldots\ldots\ldots\ldots & (a_n) \end{pmatrix}$

Cycle :

A Cycle of length r is denoted by $(a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_r)$

Where elements a_i comes from S

Like as $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$

$= (1, 4)(2, 3)$

Integral Powers :

We define $f^2 = f.f$, $f^3 = f.f.f$ $f^n = f.f.f \dots\dots f$ (n **times**)

$$f^{-1}f^{-1} = f^{-2} \quad , \quad f^{-n} = f^{-1}f^{-1} \dots\dots\dots .f^{-1} \; (n \textbf{ times})$$

But $(f.g)^m \neq f^m.g^m$, because $fg \neq gf$ in general

Order of a Permutation :

Let f be A Permutation on a finite Set S , then the order of f is the least positive integer m such that $f^m = i$, i being the identity permutation

Theorem :

The order of an r cycle is r

Like as $(1, 2, 3, 4, 5, 6) = f$ (say) in S_6 , then its order is 6

Theorem :

The order of a permutation on a finite set S is the l.c.m of the lengths of its disjoint cycles

$$\text{Let } f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 5 & 4 & 3 & 2 & 1 & 8 & 7 \end{pmatrix}$$

$$= (1, 6)(2, 5)(3, 4)(7, 8)$$

Then its order will be 2

Task : 1) find the order of $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 1 & 3 & 2 & 5 & 7 & 8 \end{pmatrix}$

Task ;

Find the inverse of $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 4 & 6 & 7 & 1 & 2 \end{pmatrix}$

Express a permutation as a product of disjoint cycles / transposition

Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 3 & 5 & 7 & 9 & 10 & 6 & 1 & 8 & 11 & 4 & 2 \end{pmatrix}$

$= (1, 3, 7)(2, 5, 10, 4, 9, 11)(6)(8)$

$= (1, 7)(1, 3)(2, 11)(2, 9)(2, 4)(2, 10)(2, 5)(6, 6)(8, 8)$

Suggested activity :

Students will involve to find out or formulate the basic structure of permutations . They will readily do the workout examples & solve some easy to moderate problems on regarding this session .

Some Problems on Permutations

1) *Describe all even permutation on the Set* $S = \{1, 2, 3, 4\}$
2) *Determine whether the followings are even or odd*

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ & & & & & & \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ & & & & & & & & \end{pmatrix}$

1 4 3 6 7 5 2 , 3 4 5 7 9 2 3 1 6

3) *Find the order of the permutation*

$$\mathcal{F} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 3 & 6 & 7 & 5 & 2 \end{pmatrix} \qquad g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & 5 & 7 & 9 & 2 & 3 & 1 & 6 \end{pmatrix}$$

Referrences ;

- *Chatterjee , B . C : Abstract algebra ,*
- *Herstein , I . N : Topics in algebra*
- *Fraleigh , John , B : Fisrt Course In abstract algebra*
- *Mapa , S . K , Higher Algebra (Abstract & Linear)*
- *Joshi , K . D : Foundations of Discrete mathematics*
- *M. K Sen : Abstract algebra*
- *Mukhopadhay < Parthasarathi , & Sen , M . K : Topics in abstract Algebra*

Session – 1

Session Name: **Limit of a Function**

Author Name: **Mrs. Rini Maiti**

Department: **Mathematics**

Subject/Course: **Theory of Real Functions**

Course Code: **CC-5**

Level of students: **B.Sc. Mathematics Hons 3rd Semester**

Cell Number: **< 8016551103>**

Session Objectives :

At the end of this session, the learner will be able to:

- Recall the definition of limit of a function
- Explain why any function can have at most one limit at a particular point
- Apply the definition of limit to find limit of a function at a particular point
- State Sequential criterion for the existence of limit
- Apply Sequential criterion for the non-existence of limit

Teaching-Learning Material:

- Black board and Chalk
- Hand written notes
- Brainstorming
- Reference Books
- Analyzing

Session Plan :

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Idea of Limit of a function	Discussions with figure drawing	Explains	Listens Participates Discusses	Remembering Understanding Interpersonal Intrapersonal Verbal-linguistic
05	Concept of existence: a function f can have at most one limit at $c \in D'$, where D is domain of f	Demonstration Discussions	Explains	Listens Discusses	Remembering Understanding Interpersonal Intrapersonal Verbal-linguistic
15	Application of definition of limit to find limit of a function	Case - study Group-Discussion	Facilitates Explains	Listens, Understands, Analyzes, Discusses, Participates	Applying Analyzing Evaluate
15	Concept of Sequential criterion for the existence of limit	Demonstration Discussions	Explains	Listens Discusses	Remembering Understanding
15	Application of Sequential criterion for the non- existence of limit	Case - study Group-Discussion	Monitors Facilitates	Solve problems Participates	Applying Analyzing Evaluate

Session Inputs :

Limit: Let *f(x)* be a given function. Then limit of *f(x)* at $x=a$ is denoted by $\lim_{x\to a} f(x)$.

(1) Left Hand Limit: Let *f(x)* be a given function. Then left hand limit of *f(x)* at $x=a$ is denoted by $\lim_{x\to a-} f(x)= \lim_{h\to 0} f(a-h)$.

(2) Right Hand Limit: Let *f(x)* be a given function. Then limit of *f(x)* at $x=a$ is denoted by $\lim_{x\to a+} f(x)= \lim_{h\to 0} f(a+h)$.

Existence of limit at a point: Let *f(x)* be a given function. If $\lim_{h\to 0} f(a-h) = lim_{h\to 0} f(a+h)$, then limit of *f(x)* at $x=a$ exists and $lim_{h\to 0} f(a-h) = lim_{h\to 0} f(a+h) = lim_{x\to a} f(x)$.

If $lim_{h\to 0} f(a-h) \neq lim_{h\to 0} f(a+h)$, then limit of *f(x)* does not exist at $x=a$.

Explanation by example: $\lim_{x\to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$ does not exist.

<u>Solution:</u> $\lim_{x\to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1} = lim_{x\to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}[1+e^{-1/x}]} = \lim_{x\to 0} \frac{1}{1+e^{-1/x}}$.

LHL: $\lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{1}{1+e^{1/h}} = 0.$

RHL: $\lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{1}{1+e^{-1/h}} = 1.$

Therefore LHL≠RHL. So $\lim_{x\to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$ does not exist.

Sequential Criterion: Let D be a subset of R and f: D→R be a function. Let a be a limit point of D and $l \in$R. Then $\lim_{x\to a} f(x)=l$ if and only if for every sequence $\{x_n\}$ in D$-\{a\}$ converging to a, the sequence $\{f(x_n)\}$ converges to l .

Suggested Activity:

Students are involving to understand & calculate left hand limit and right hand limit.

They are trying to check whether limit of a function at a point exists or not.

They are trying to apply Sequential criterion of limit for proving non-existence of limit of a function at a point.

Simple problems on limit:

Problem1: If $f(x) = \begin{cases} x & if\ x \in Q \\ -x & if\ x \in R-Q \end{cases}$. Prove that $\lim_{x\to a} f(x)$ exists if a=0.

.Problem 2: If $f(x) = \begin{cases} ax+1 & if\ x > 1 \\ x & if\ x < 1 \\ 1 & if\ x = 1 \end{cases}$ and $\lim_{x\to 1} f(x)$ exists, then find the value of a.

Problem 3: Find $\lim_{x\to 0}(1+x)^{1/x}$.

Summary:

In this session, we learnt

- How to find left hand limit of a function at a given point.
- How to find right hand limit of a function at a given point.
- How to find limit of a function at a given point.
- Application of Sequential Criterion.
- Existence & non-existence of imit of a function at a given point.

Assignment :

- Let D be a subset of R and f : D→ R *be* a function. Let c be a limit point of D. Then prove that f can have at most one limit at c.
- Prove that $\lim_{x\to 0} f(x)$ does not exist, where $f(x)=sin\frac{1}{x}$, $x\neq 0$.
- Show that $\lim\limits_{x\to 0}[x]$ does not exist.
- Show that $\lim\limits_{x\to 0}[x]$ does not exist.

References:

- Introduction to Real Analysis - S. K. Mapa
- Mathematical Analysis – S. C. Malik & Savita Arora
- Principles of Mathematical Analysis – Walter Rudin
- Elements of Real Analysis – M. D. Raisinghania, Santi Narayan

SESSION PLAN OF CC 6

Session – 1 (2022-2023) (Odd Semester)

Session Name : Integral Domain & Field

Teacher Name : Mr. Snehendu Mandal

Department : Mathematics

Subject / Course : Ring Theory-I

Course Code : CC 6

Level Of Students : B.Sc. Math (Hons) , 3^{rd} Sem

Contact No : < 8436475960 >

Session objectives :

At the end of the session student will be able to

- Understand the definition of Integral Domain
- Several examples of integral domain
- When (Z_n , + .) becomes an Integral domain
- Know about Gaussian integers Z [i] as an Integral Domain
- Know about Z [x] as an Integral Domain
- Characteristic of an Integral domain
- Understand about Skew Field
- Undersatnd the basics about Field
- Several examples about Field

Teaching –Learning Material

- Black Board & Chalk
- Brainstorming
- Referrence Book
- Notes
- Pen & Paper
- White Board & Marker

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Session Plan

Time (in min)	Content	Learning aid & Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardners)
05	Review of Basics about Ring theory	Brainstorming Case Study	Explains, Demonstrate Question Answer	Listens writing	Remembering Understanding
10	Idea about Integral Domain	Brainstorming & Explains	Expains Demonstrate Examples	Listens Discuss Analyze	Understanding Analyzing Applying Participates
10	Know about Characteristic of Integral Domain , Definition of skew Field	Case -Study Formulate Analyze	Explains & Facilitates Demonstrate	Analyze Understand Participate	Analyzing Understanding Applying
10	Finite Integral domain & Field	Brainstorming Demonstration	Deductions & Explains	Listens Participates Discuss	Remembering Visual - spatial Intrapersonal
10	Several Examples about Finite Field	Case -Study Group discussion	Demonstrate Analyze	Formulate Analyze	Understanding Intrapersonal
15	Various Problems on Integral Domain & Fiel d	Innovative Conclusion Group discussion	Hints for Solution	Participates Group - Discuss	Conceptual Approach Remembering Applying knowledge to solve problems

Session Inputs :

Definition of Integral Domain :

A Non – trivial ring R with unity is said to be an Integral domain if R is
Examples of Integral Domain ;

- ***(Z , + , .) ; (R , + .) ; (Q , + , .)*** *are all integral domains*
- ***(2Z , + , .) (3Z , + , .) ……… (mZ , + , .)*** *are all integral domain*
- *(Z_5 ,+ , .) is integral domain as it contains no divisor of zero also it is Commutative*
- *(Z_6 ,+ , .) is not an integral domain as it contains Divisors of zero Because [2].[3] = [0] , [2] , [3] are divisors of zero*
- *The ring (M_n ,+ , .) is not integral domain as it is non- commutative*
- *The ring Z× Z is not integral domain as (1, 0) .(0 , 1) = 0 , so it contains divisor of zero*
- *Similarly $Z \times Z \times Z$, $Z \times Z \times Z \times Z$, $Z \times Z \times Z \times Z \times Z$, ……….. $Z \times Z \times Z \times Z \times Z \times$ ……….$\times Z$ all are not an integral domain*

A very useful result about Z_n is that

(Z_n , + , .) will be an integral domain iff n is a prime

We first show two an examples

Consider (Z_8 ,+ , .) & (Z_7 ,+ , .)

*In (Z_8 ,+ , .) we seen that **[2].[4] = [0]** , so we have divisor of zero , hence it is not an integral domain*

But in (Z_7 ,+ , .) , there is no divisor of zero , there does not exist any m & n such that [m].[n] = [0] , hence it is an integral domain

- ***Prove that the ring of Gaussian integer Z[i] is an integral domain***

Z[i] = { a+ib : a , b ϵ Z }

This ring is ring with unity 1 , also it is commutative

We just need to check it contains any divisors of zero

Let (a+ib) . (c+id) = 0 that will imply

$ac - bd = 0$, $ad + bc = 0$ *it will give c = 0 & d = 0*

hence it contains no divisor of zero , so it is an integral Domain

- ***Prove that Z [x] is an integral domain***

$\boldsymbol{Z\,[\,x\,] = \{\ a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots . a_n x^n}$: a_i *are integers & x is an indeterminate* }

Let f(x) & g (x) be two polynomials in Z [x]

$\boldsymbol{\mathcal{F}(x) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots . a_n x^n}$ ***let*** $\boldsymbol{a_n \neq 0}$

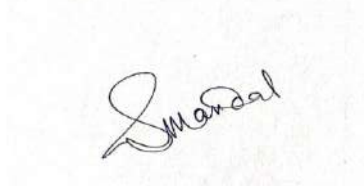
$\boldsymbol{G(x) = b_0 + bx + b_2 x^2 + \ldots\ldots\ldots\ldots . . b_m x^m}$ ***let*** $\boldsymbol{b_m \neq 0}$

Then f(x).g(x) ≠ 0 for non – zero polynomials f(x) & g(x)

Hence we say that Z [x] is an integral domain.

- *The Characteristic of an integral domain is zero or prime*
- *Skew Field / Division Ring : A non- trivial ring R with unity is said to be a division ring if every non – zero is a unit*
- *Task : Prove The ring of Rational quaternions is a skew field*

$$\text{Let } \mathcal{P} = \left\{ \begin{pmatrix} a+ib & c+id \\ -c+id & a+ib \end{pmatrix} \quad a, b, c, d \in Q \right\}$$



Suggested activity :

Students will involve to find out or formulate the basic structure ofIntegral Domains & Finite Field . They will readily do the workout examples & solve some easy to moderate problems on regarding this session .

Some Problems on Permutations

- Prove that the set $\{ m+in : m , n \in R \}$ is a field
- Prove that the set $\{ m+in : m , n \in Q \}$ is a field
- Prove that the set $\{ a+i\sqrt{b} : m , n \in Q \}$ is a field

Prove that the ring of matrices $\left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a , b \in R \right\}$

is a field .

- Prove that the ring of matrices $\left\{ \begin{bmatrix} a & b \\ 2b & a \end{bmatrix} : a , b \in Q \right\}$

is a field .

- Show that the Set $Z[\sqrt{-5}] = \{ a + ib\sqrt{5} : a , b \in Z \}$ is an integral domain
- Find all the units in the integral domain $Z[i]$
- Find all the units in the integral domain $Z[\sqrt{-7}]$
- Find all the units in the integral domain $Z_m \times Z_n$, where m , n are co-prime
- Find all the units in the integral domain $Z_m \times Z_n \times Z[i]$

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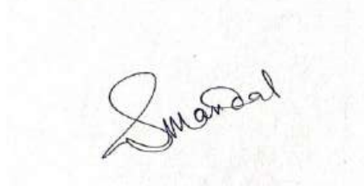
Referrences ;

- *Herstein , I . N : Topics in algebra*
- *Mapa , S . K , Higher Algebra (Abstract & Linear)*
- *M. K Sen : Abstract algebra*
- *Mukhopadhay , Parthasarathi , & Sen , M . K : Topics in abstract Algebra*
- *Whitelaw , T .A : Introduction to Abstract Algebra .*



Session – 30

Session Name: **Concepts of Neighbourhood, Open Set, Closed Set**

Author Name: **Mrs. Rini Maiti**

Department: **Mathematics**

Subject/Course: **ODE & Multivariate Calculus-1**

Course Code: **CC-7**

Level of students: **B.Sc. Mathematics Hons 3rd Semester**

Cell Number: **< 8016551103>**

Session Objectives :

At the end of this session, the learner will be able to:

- Recall the definition of open n-ball in R^n, neighbourhood, interior point, open set, closed set
- Find the set of all interior points of any set in R^n
- Explain why Φ & R^n are open sets in R^n, every open n-ball is an open set
- Check whether a set in R^n is an open set or a closed set

Teaching-Learning Material:

- White board and Marker
- Brainstorming
- Reference Books
- Hand-written Notes
- Analyzing

Session Plan :

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	The set R^n & the distance in it	Discussions with board work	Explains	Listens Participates Discusses	Understanding Interpersonal Intrapersonal Verbal-linguistic
15	Concept of open n-ball in R^n, circular, rectangular & square neighbourhood	Discussions with figure drawing & by giving various examples taking $n = 1,2,3$	Explains	Listens Participates Discusses	Remembering Understanding Analyzing Interpersonal Intrapersonal Verbal-linguistic
15	Concept of neighbourhood, interior point, open set & closed set	Discussions by giving various examples. Case - study Group-Discussion	Facilitates Explains	Listens, Understands, Analyzes, Discusses, Participates	Applying Analyzing
20	Proof of some important theorems regarding open sets & closed sets	Demonstration Discussions	Explains	Listens, Understands, Discusses	Remembering Understanding

Session Inputs :

The set R^n & the distance in it:

R^n denotes the set of ordered n-tuples ($x_1, x_2, \dots \dots \dots, x_n$) of real numbers $x_i, i = 1,2, \dots \dots \dots, n$.

The distance between the points $x = (x_1, x_2, \dots \dots \dots, x_n)$, $y = (y_1, y_2, \dots \dots \dots, y_n) \in R^n$ is defined as $d(x, y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}$.

Note: (i) If $x = (x_1, x_2, \dots \dots \dots, x_n) \in R^n$, then x_i is called the i-th component of x.

(ii) R^n is called n-dimensional Euclidean space.

(iii) (a) $d(x, y) \geq 0$

(b) $d(x, y) = 0 \Leftrightarrow x = y$

(c) $d(x, y) = d(y, x)$

(d) $d(x, z) \leq d(x, y) + d(y, z)$

Open ball in R^n / Open n- ball in R^n :

Let $a \in R^n$ & $\delta > 0$. Then the set $B(a, \delta) = \{ x \in R^n : d(x, a) < \delta \}$ is called the open ball of radius δ & centre at a.

$B(a, \delta)$ is also called the δ-nbd of the point $a \in R^n$.

Examples:

1) In R^1 i.e. in R, $B(a, \delta)$ is the open interval

$|x - a| < \delta \Rightarrow -\delta < x - a < \delta \Rightarrow a - \delta < x < a + \delta$ i.e. $B(a, \delta) = (a - \delta, a + \delta)$.

2) In R^2, $B(a, \delta)$ is a circular disc of radius δ & centre a. $B(a, \delta)$ is called a circular δ-nbd of $a \in R^2$.

Let $a = (a_1, a_2)$, $x = (x_1, x_2) \in R^2$.

Then d(x, a) $< \delta \Rightarrow \sqrt{(x_1 - a_1)^2 + (x_2 - a_2)^2} < \delta$

$$\Rightarrow (x_1 - a_1)^2 + (x_2 - a_2)^2 < \delta^{\,2}.$$

i.e. the set of all points $(x_1, x_2) \in R^2$ that lie within the circle

$$(x_1 - a_1)^2 + (x_2 - a_2)^2 = \delta^{\,2}.$$

3) In R^3, B(a, δ) is a spherical solid of radius δ & centre at a. B(a, δ) is called a spherical δ-nbd of a$\in R^3$.

Let a = (a_1, a_2, a_3), $x = (x_1, x_2, x_3) \in R^3$.

Then d(x,a) $< \delta \Rightarrow \sqrt{(x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 - a_3)^2} < \delta$

$$\Rightarrow (x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 - a_3)^2 < \delta^{\,2}.$$

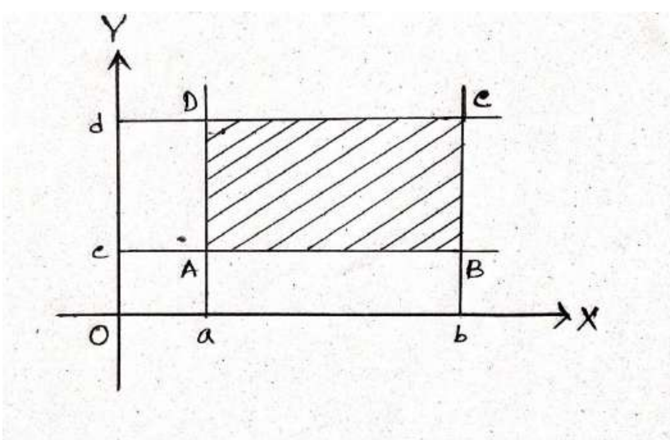
i.e. the set of all points $(x_1, x_2, x_3) \in R^3$ that lie within the sphere

$$(x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 - a_3)^2 = \delta^{\,2}.$$

Note**:** In R^2 , we also use other nbd.

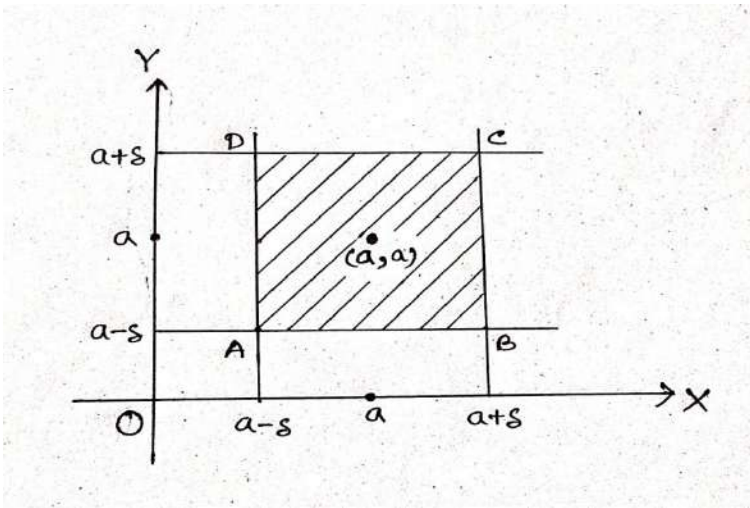
For example-

1) Rectangular nbd: (x,y) $\in R^2$ s.t. a < x < b, c < y < d



2) Square nbd: (x,y) $\in R^2$ s.t. $|x-a| < \delta \Rightarrow -\delta < x-a < \delta \Rightarrow a-\delta < x < a+\delta$

& $|y-a| < \delta \Rightarrow -\delta < y-a < \delta \Rightarrow a-\delta < y < a+\delta$.



Neighbourhood:

Let $S \subset R^n$ & $a = (a_1, a_2, \ldots \ldots \ldots, a_n) \in R^n$. Then S is said to be a nbd of a$\in R^n$ if $\exists\ \delta > 0$ s.t. $B(a, \delta) = \{x = (x_1, x_2, \ldots \ldots \ldots, x_n) \in R^n : d(x,a) < \delta\} \subset S$ i.e. $B(a, \delta)$ is

$\{x = (x_1, x_2, \ldots \ldots \ldots, x_n) \in R^n : \sqrt{(x_1 - a_1)^2 + (x_2 - a_2)^2 + \cdots + (x_n - a_n)^2} < \delta\}$.

Interior point:

Let $S \subset R^n$. A point $a \in S$ is said to be an interior point of S if $\exists\ \delta > 0$ s.t. $B(a, \delta) \subset S$, where $B(a, \delta)$ denotes the open n-ball of radius δ & centre at a.

The set of all interior points of S is called the interior of S & it is denoted by int S.

Open Set:

A set $S \subset R^n$ is said to be an open set in R^n if for all a$\in$S, $\exists\ \delta > 0$ s.t.

$B(a, \delta) \subset S$, where $B(a, \delta)$ denotes the open n-ball of radius δ & centre at a.

Examples: (i) $\emptyset$ is an open set in R^n.

(ii) R^n an open set in R^n.

(iii) Every open n-ball is an open set.

Closed Set:

A set S $\subset R^n$ is said to be a closed set in R^n if its complement R^n −S is open in R^n.

Example: The set $\bar{B}(a,\delta) = \{x = (x_{1,}x_2, \ldots \ldots \ldots, x_n) \in\ R^n :\ d(x,a) <\ \delta\}, \delta > 0$, is a closed set in R^n.

Theorems:

1) Union of any family open sets in R^n is an open set.

2) The intersection of a finite number of open sets in R^n is an open set.

3) The union of a finite number of closed sets in R^n is a closed set in R^n.

4) The intersection of any family of closed sets in R^n is a closed set in R^n.

Suggested Activity:

Students are involving to understand different kind of neighbourhoods.

They are trying to define open n-ball in R^n for different values of n. They are trying to check whether a set is an open set, a closed set.

Simple Problems:

Problem1: Examine whether the following sets ($\subset R^2$) are open or not:

(i) $\{(x, y): x^2 + y^2 < 1\}$ (ii) $\{(x, y): xy < 1\}$

Problem 2: Examine whether the following sets ($\subset R^2$) are closed or not:

(i) $\{(x, y): 2x + 3y \geq 4\}$ (ii) $\{(x, y): 0 < x < 1, 0 \leq y \leq 1\}$

Problem 3: If A be the set of all points (x, y) such that both x & y are rational, the set A is neither open nor closed.

Summary:

In this session, we learnt

- How to calculate the distance between two points in R^n.
- Different kinds of neighbourhoods.
- To examine whether a set is open or closed.

Assignment:

- Show that $G = \{(x, y) \in R^2 : x > 0, y > 0\}$ is an open set.
- Examine whether the following sets ($\subset R^2$) are open or not:
  (i) $\{(x, y): x + y < 1\}$ (ii) $\{(x, y): xy \leq 1\}$
- The set consists of all points (x, y)such that $y = sin\frac{1}{x}$ and $x > 0$. Does this set have any interior point ? Is it closed ? .
- Correct or justify: The set made up of all points of the x-axis together with all points of the y-axis is closed.

References:

- Advanced Differential Calculus of Several Variables – Subir Kumar Mukherjee
- Introduction to Differential Calculus – Ghosh & Maity
- A Course of Mathematical Analysis – Shanti Narayan & P.K.Mittal
- Calculus – Apostole
- Mathematical Analysis – S.C.Malik & Savita Arora

Session – 1

Session Name: Looping in C

Author Name: Dr. Arun Kumar Maiti

Department: Mathematics

Subject/Course: C Programming Language

Course Code: SECA

Level of students: B.Sc. Mathematics(Hons),1st Sem.

Cell Number:< 9434568316>

Session Objectives

At the end of this session, the learner will be able to:

- Learn about looping in C.
- Get idea about different types of loops.
- Construct Flow Charts of different loops
- Solve Various types of problems using loop.

Teaching Learning Material

- Brainstorming
- Presentation slides
- Black Board and Chalk
- Game

Session Plan

Time (in min)	**Content**	**Learning Aid and Methodology**	**Faculty Approach**	**Typical Student Activity**	**Learning Outcomes (Blooms + Gardeners)**
05	Review of previous lesson	Brainstorming	Question Answer	Listens Participates Discusses	Remembering Understanding
10	Define loops and its necessity	Brainstorming	Facilitates Explains	Listens Watches Discuss	Understanding
10	For loop , its syntax and flowchart	Demonstration	Explains	Listens Analyzes	Analyzing Intrapersonal Logical Linguistic
10	While loop, its syntax and flowchart	Demonstration Discussions	Deduction Analyze	Listens Participates Analyzes Discusses	Remembering Understanding Interpersonal Intrapersonal Visual-spatial Logical
10	Do-While loop, its syntax and flowchart	Group Discussions	Explains	Listens Analyzes	Remembering Understanding
15	Solve Simples problems using for, while and do-while loops	Innovative conclusion	Hints for solution	Participates	Remembering Understanding Applying Knowledge to Solve Problems

Session Inputs

Loops and its uses :

Iterations or loops are used when we want to execute a statement or block of statements several times. The repetition of loops is controlled with the help of a test condition. The statements in the loop keep on executing repetitively until the test condition becomes false.

The loops in C language are used to execute a block of code or a part of the program several times. In other words, it iterates/repeat a code or group of code many times.

A looping process would include the following four steps.

1 : Initialization of a condition variable.
2 : Test the condition.
3 : Executing the body of the loop depending on the condition.
4 : Updating the condition variable.

C language provides three iterative/repetitive loops.

1 : **for** loop 2 : **while** loop 3: **do-while** loop

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

If I ask What are the necessities of loops? Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting 5 responses, we will have a close look at all the points and remove irrelevant points with proper explanations. From the remaining responses the correct answer can be frame as follows:

1. Repetition: Loops enables us to perform a specific task repeatedly without writing duplicate code.
2. Efficient code: Loops reduce code size and make it more efficient by eliminating the need for repetitive code.
3. Dynamic Data Processing: Loops allow us to process dynamic data, such as arrays, structures and linked lists.

4. Algorithm Implementation: Loops are essential for implementing various algorithms, like sorting ,searching and Mathematical calculations.

For Loop, its Syntax and its uses:

Notes

For Loop:

- This is an entry controlled looping statement.
- In this loop structure, more than one variable can be initialized.
- One of the most important features of this loop is that the three actions can be taken at a time like variable initialization, condition checking and increment/decrement.
- The for loop can be more concise and flexible than that of while and do-while loops.

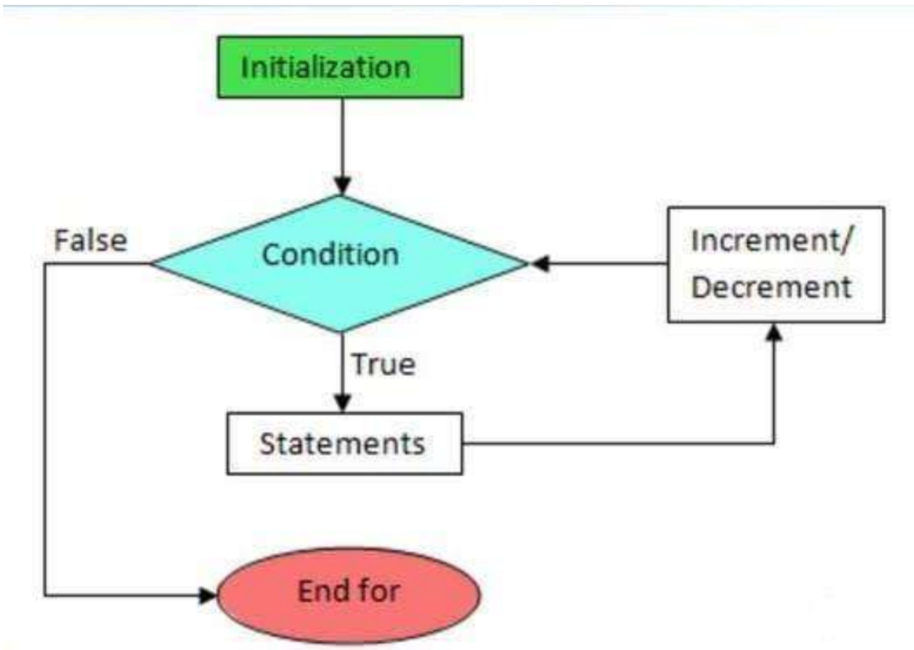
For loop :- The general **syntax** of for loop consists of three expressions separated by semicolons.
It is given as follows:-

```
for(expression1;expression2;expression3)
Body of loop condition Next statement out of loop
 {
Statement(s);
}
```

Here the **expression1** is the initialization expression,**expression2** is the test expression and **expression3** is the update expression. **Expression1** is executed only once when the loop starts and is used to initialize the loop variables.
Expression2 is a condition and is tested before each iteration of the loop. Expression3 is an update expression and is executed each time after the body of the loop is executed.

Syntax : for(initialization; condition; increment/decrement)

The Flowchart of for loop:



Suggested Activity:

We can conduct brainstorming activity by posing following question to the student.

If I ask why is for loop an entry controlled statement? Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting few responses, we will have a close look at all the points and remove irrelevant points with proper explanations. The correct answer can be put as follows :

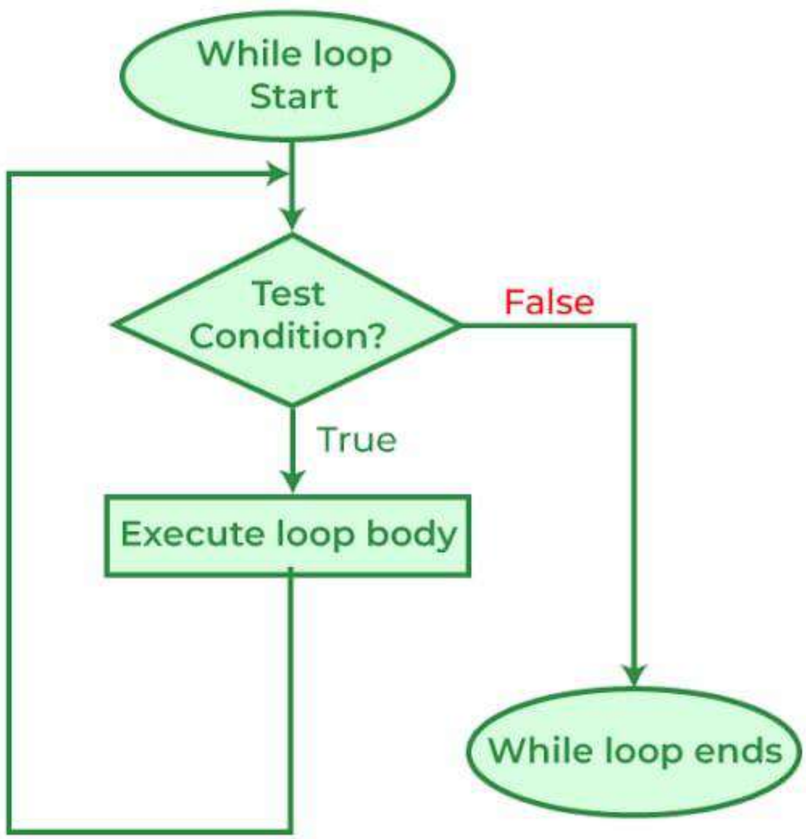
In a For loop, the condition is evaluated before the loop body is executed. If the condition is true, the loop body is executed, and then the increment/decrement operation is performed. If the condition is false, the loop is terminated. That is why it is an entry controlled loop.

While Loop, its Syntax and its uses:

A **while** loop is a control flow statement in programming that allows you to execute a block of code repeatedly while a certain condition is true.
It is the fundamental looping statement in C. It is suited for the problems where it is not known in advance that how many times a statement or block of statements will be executed. The general syntax of while loop is as under:-

```
While(condition)
{
Statement(s);
}
```

We explain the working of while loop with the help of flow chart



Suggested Activity:

We can conduct brainstorming activity by posing following question to the student.

If I ask why is while loop an entry controlled statement? Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting few responses, we will have a close look at all the points and remove irrelevant points with proper explanations. The correct answer can be put as follows :

Announcement

The while loop is an entry controlled loop statement, i.e means the condition is evaluated first and it is true, then the body of the loop is executed. After executing the body of the loop, the condition is once again evaluated and if it is true, the body is executed once again, the process of repeated execution of the loop continues until the condition finally becomes false and the control is transferred out of the loop.

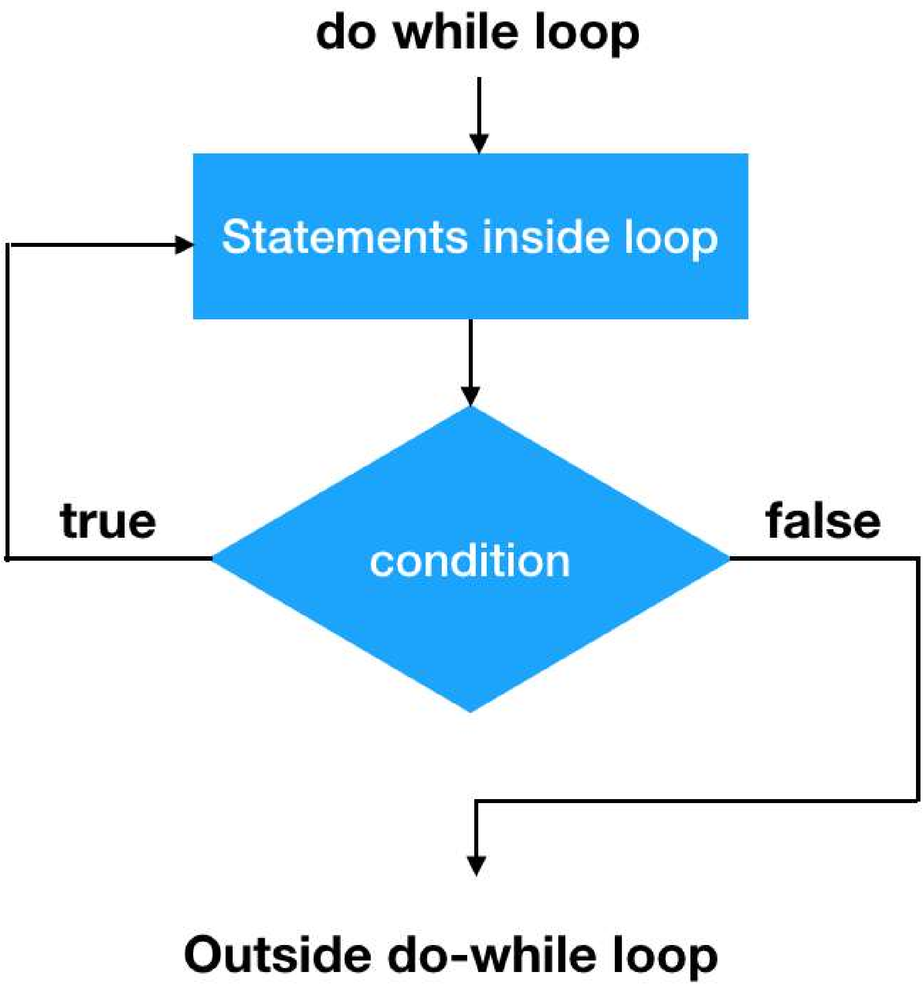
Do-While Loop, its Syntax and its uses:

Notes

The **do-while** loop is an exit controlled loop statement The body of the loop are executed first and then the condition is evaluated. If it is true, then the body of the loop is executed once again. The process of execution of body of the loop is continued until the condition finally becomes false and the control is transferred to the statement immediately after the loop. The statements are always executed at least once.

do-while loop Syntax :
variable initialization ;
do
{
statements ; variable increment or decrement ;
}**while** (condition);

The Flowchart of do-while loop:



Suggested Activity:

We can conduct brainstorming activity by posing following question to the student.

If I ask what are the differences between while loop and do-while ? Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.
After getting few responses, we will have a close look at all the points and remove irrelevant points with proper explanations. The correct answer can be put as follows :

Difference between while and do-while loop :

While loop is pre test loop.	do-while is post test loop.
The statements of while loop may not be executed even once.	The statements of do-while loop are executed at least once.
There is no semi colon given after while(condition).	There is semi colon given after while(condition);

The syntax of while loop is While(condition)
{
Statements
}
	The syntax of do-while loop is as under:- Do
	{
	Statements;
	}while(condition);

Simple Problems using loops:

Problem 1: Write a c-program to generate Fibonacci series using for loop

```
/* program to generate the Fibonacci series*/
 #include <stdio.h>
 void main()
{
int x,y,z;
 int i,n;
 x=0; y=1;
 printf("enter the number of terms");
scanf("%d",&n);
printf("%d",y);
for(i=1;i<n;i++)
{
z=x+y;
printf("%d",z);
x=y;
y=z;
 }
```

```
printf("\n");
getch();
}
```

Problem 2: Write a c-program to generate Fibonacci series using while loop

```
/ *Fibonacci series in C using while loop*/
#include <stdio.h>
int main()
{
int n1 = 0, n2 = 1, n3, count;
printf("Enter the limit \n");
scanf("%d", &count);
printf("\n%d\n%d\n", n1, n2);
count = count - 2;
while(count)
{
n3 = n1 + n2;
printf("%d\n", n3);
n1 = n2;
n2 = n3;
count = count - 1;
}
return 0
}
```

Problem 1: Write a c-program to generate Fibonacci series using do-while loop

```
    /* program to generate the Fibonacci series*/

#include <stdio.h>

int main()
 {
   int n = 10, i = 2, fib1 = 0, fib2 = 1, nextTerm;

   printf("Fibonacci Series: %d, %d, ", fib1, fib2);

   do {
```

```
            nextTerm = fib1 + fib2;
            printf("%d, ", nextTerm);
            fib1 = fib2;
            fib2 = nextTerm;
            i++;
        } while (i < n);

        return 0;
    }
```

Suggested Activity:

By solving the above problems learners will be able to write c-program to generate Fibonacci series using different loops.

Now for better understanding I ask a question to the Students :

Write a c program to find the sum of the series : 1 + 1/2 + 1/3 + ... + 1/N

Students write down the c-program as follows:

```
/ * C Program to find the Sum of Series 1 + 1/2 + 1/3  + ... + 1/N */

#include <stdio.h>

 void main()

{

double number, sum = 0, i;

     printf("\n enter the number ");

    scanf("%lf", &number);

    for (i = 1; i <= number; i++)

    {
```

```
            sum = sum + (1 / i);
            if (i == 1)
                printf("\n 1 +");
            else if (i == number)
                printf(" (1 / %lf)", i);
            else
                printf(" (1 / %lf) + ", i);
        }
        printf("\n The sum of the given series is %.2lf", sum);
    }
```

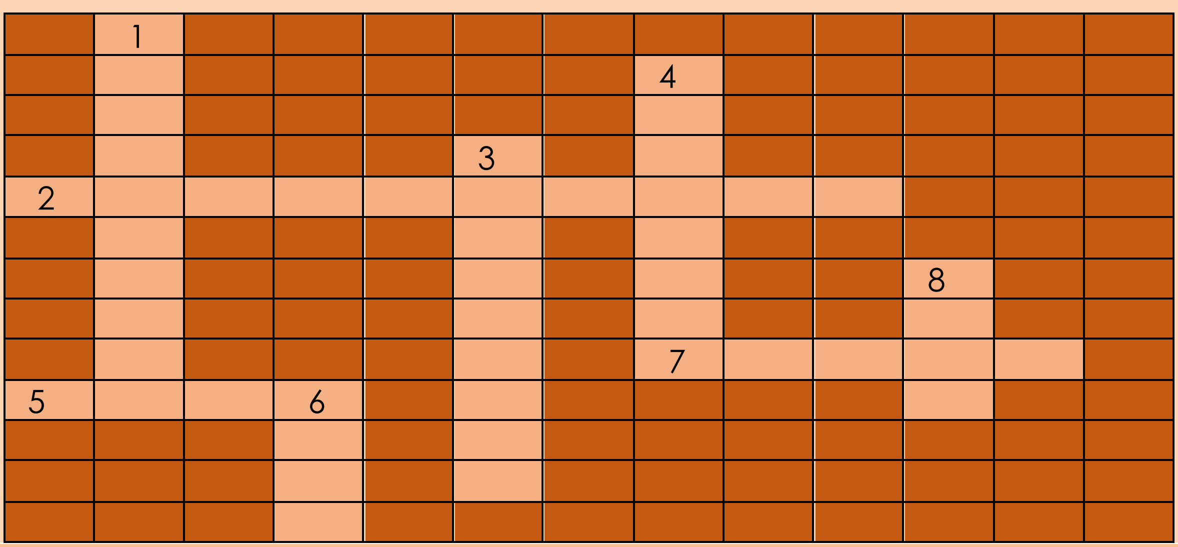
Conclusion:

The session can be concluded with a little GAME (Fastest Hand raising when you solve a cross-word puzzle)

Suggested Activity:

The following crossword puzzle will be placed in front of students to answer with a game Fastest Hand raising.

Prize: The winner with maximum answers gets a chocolate.



Clues:

Sl. No.	Across:	Sl. No.	Down
2	If test expression is false then while loop………..	1	2nd step of for loop is checking last…………
5	While loop executes at least………	3	3rd step of for loop is…………
7	For loop is…………..controlled loop.	4	Loops are used to execute a block of code…………times.
		6	Do-while loop is ……….controlled loop.
		8	The loop works as long as the input number is not…………..

Answer to the Crossword Puzzle:

	E 1											
	X						M 4					
	P						U					
	R				I 3		L					
T 2	E	R	M	I	N	A	T	E	S			
	S				C		I					
	S				R		P			Z 8		
	I				E		L			E		
	O				M		E 7	N	T	R	Y	
O 5	N	C	E 6		E					O		
			X		N							
			I		T							
			T									

Summary

In this session, we learnt:

1. About the looping in c.
2. About the differences between the loops.
3. How to write c-programs for different types of problems using loops.

Assignment

1. Write a c program to find roots of a quadratic equation
2. Write a c program to check whether a number is prime or not.
3. Write a c program to find the sum of the series 1+2+3+………+N.
4. Write a c program for sorting an array of numbers.

References

1. Computing Fundamentals and C Programing – E.Balagurusamy
2. Let Us C – Y. Kanetkar
3. C In Depth – Srivastava and Srivatava

Session – 1

Session Name: **Riemann Integration**

Author Name: **Mrs. Rini Maiti**

Department: **Mathematics**

Subject/Course: **Riemann Integration & Series of Functions**

Course Code: **CC-8**

Level of students: **B.Sc. Mathematics Hons 4th Semester**

Cell Number: **< 8016551103>**

Session Objectives :

At the end of this session, the learner will be able to:

- ❖ Recall the definition of partition, upper Darboux sum, lower Darboux sum
- ❖ Find the lower integral, upper integral of a function
- ❖ Explain when a function will be Riemann integrable

Teaching-Learning Material:

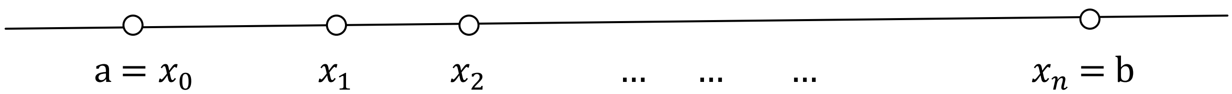
- ❖ Black board and Chalk
- ❖ Brainstorming
- ❖ Hand-written Notes
- ❖ Reference Books
- ❖ Analyzing

Session Plan :

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Idea of a partition & sub-intervals	Discussions with figure drawing	Explains	Listens Participates	Understanding
15	Concept of Upper Darboux Sum & Lower Darboux Sum	Discussions by giving various examples	Explains	Listens Participates Discusses	Remembering Understanding Analyzing Interpersonal Intrapersonal Verbal-linguistic
15	Concept of Lower Integral & Upper Integral	Discussions by giving various examples. Case - study Group-Discussion	Facilitates Explains	Listens, Understands, Analyzes, Discusses, Participates	Applying Analyzing
20	Concept of Riemann integrable functions	Demonstration Discussions by giving various examples	Explains	Listens, Understands, Discusses	Remembering Understanding Applying Analyzing

Session Inputs :

Partition of a Closed Interval: Let I =[a, b] be a closed and bounded interval. A partition of [a, b] is a finite ordered set P= $(x_0, x_1, \ldots\ldots, x_n)$ of points of [a, b] such that a = $x_0 < x_1 < x_2 < \ldots\ldots < x_n$ = b.



Example: P = $(0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1)$ is a partition of [0, 1]. Q = $(0, \frac{1}{4}, \frac{3}{8}, \frac{2}{4}, \frac{3}{4}, \frac{7}{8}, 1)$ is another partition of [0, 1].

Note: The family of all partitions of [a, b] is denoted by P[a, b].

Sub-intervals: Let I =[0, 1]. Then $I_1=[0, \frac{1}{4}]$, $I_2=[\frac{1}{4}, \frac{2}{4}]$, $I_3=[\frac{2}{4}, \frac{3}{4}]$, $I_4=[\frac{3}{4}, 1]$ are called sub-intervals of I =[0, 1].

Riemann-integrability:

Let I =[a, b] be a closed and bounded interval. Let f : [a, b]→R be a bounded function on [a, b]. Let us take a partition P= $(x_0, x_1, \ldots\ldots, x_n)$ of [a, b], where a = $x_0 < x_1 < x_2 < \ldots\ldots < x_n$ = b. Since f is bounded on [a, b], f is bounded on $[x_{r-1}, x_r]$, for r = 1, 2,3, …….. , n.

Let M = $\sup_{x\in[a,b]} f(x)$, m = $\inf_{x\in[a,b]} f(x)$;

$$M_r = sup_{x\in[x_{r-1},x_r]} f(x), \; m_r = inf_{x\in[x_{r-1},x_r]} f(x), \text{ for } r = 1, 2,3, \ldots\ldots, n.$$

Then m ≤ $m_r \le M_r$ ≤ M , for r = 1, 2,3, …….. , n.

The sum $M_1(x_1 - x_0) + M_2(x_2 - x_1) + \ldots \ldots \ldots + M_n(x_n - x_{n-1})$ is said to be the Upper Darboux Sum of f corresponding to the partition P and is denoted by U(P, f).

Example: Let f(x) = x, x∈[0, 4] & P = (0, 3, 4), I_1=[0, 3] , I_2=[3, 4].

Let $M_1 = sup_{x\in[0,3]} f(x)$, $M_2 = sup_{x\in[3,4]} f(x)$.

Then $M_1 = 3$, M_2 =4 and U(P, f) = $M_1(3-0) + M_2(4-3)$

= 3×3 + 4×1 = 9 + 4 = 13.

And the sum $m_1(x_1 - x_0) + m_2(x_2 - x_1) + \ldots \ldots \ldots + m_n(x_n - x_{n-1})$ is said Lower Darboux Sum of f corresponding to the partition P and is denoted by L(P, f).

Example: Let f(x) = x, x∈[0, 4] & P = (0, 3, 4), I_1=[0, 3] , I_2=[3, 4].

Let $m_1 = inf_{x\in[0,3]} f(x)$, $m_2 = inf_{x\in[3,4]} f(x)$.

Then $m_1 = 0$, m_2 =3 and L(P, f) = $m_1(3-0) + m_2(4-3)$

= 0×3 + 3×1 = 3.

The supremum of the set {L(P, f) : P∈ $P\ [a,b]$ } is called the Lower Integral of f on [a, b] and is denoted by $\int_{-a}^{b} f(x)dx$.

The infimum of the set {U(P, f) : P∈ $P\ [a,b]$ } is called the Upper Integral of f on [a, b] and is denoted by $\int_{a}^{-b} f(x)dx$.

f is said to be Riemann integrable on [a, b] if $\int_{-a}^{b} f(x)dx = \int_{a}^{-b} f(x)dx$. And the common value $\int_{-a}^{b} f(x)dx\ or\ \int_{a}^{-b} f(x)dx$ is denoted by $\int_{a}^{b} f(x)dx$.

Example: (i) Let I =[a, b] be a closed and bounded interval and c∈R. A function f : [a, b]→R is defined by f(x) = c , x∈ $[a, b]$. Then f is Riemann integrable on [a, b].

(ii) A function f : [0, 1]→R is defined by f(x) $\begin{cases} 1, if\ x\ is\ rational \\ 0, if\ x\ is\ irrational \end{cases}$.

Then f is not Riemann integrable on [0, 1].

Suggested Activity:

Students are involving to understand partition of a closed and bounded interval & sub-intervals.

They are trying to calculate the Upper Darboux Sum & Lower Darboux Sum, Lower Integral & Upper Integral. They are trying to check whether a function is Riemann integrable on a given interval.

Simple Problems on Mapping:

Problem1: Find three different partitions of [0, 5].

Problem 2: Find the Upper Darboux Sum & Lower Darboux Sum of the function f : [0, 1]→R, which is defined by f(x) = $\begin{cases} 1, if\ x\ is\ rational \\ -1, if\ x\ is\ irrational \end{cases}$.

Problem 3: Find the Lower Integral & Upper Integral of the function f : [0, 1]→R, which is defined by f(x) = x, x∈R.

Problem 4: Give an example of a non-Riemann integrable function on [0, 2].

Summary:

In this session, we learnt

- How to find partition of a closed and bounded interval & sub-intervals.
- How to calculate Upper Darboux Sum & Lower Darboux Sum of a function on a given interval.
- To calculate the Lower Integral & Upper Integral of a function on a given interval.
- To explain with proper justification why a function is Riemann integrable on a given interval or not.

Assignment:

- Let f : N→R be defined by $f(n) = \frac{n^2+1}{n^2}$, $n \in$ R. Find the range of the sequence $\{ f(n)\}$.
- Let f : N→R be defined by $f(n) = \frac{(-1)^n}{n}$, $n \in$ R. Find the limit of the sequence $\{ f(n)\}$.
- Let f : N→R be defined by $f(n) = \frac{3n^2+2n+1}{n^2+1}$, $n \in$ R. Prove that the limit of the sequences $\{ f(n)\}$ is 3.
- Give an example of a sequence which is bounded above but unbounded below.
- Give an example of a sequence which is unbounded above but bounded below.

References:

- Introduction to Real Analysis – S.K.Mapa
- Mathematical Analysis – S.C.Malik & Savita Arora
- Introduction to Real Analysis – Bartle & Sherbert
- Real Analysis – N.P.Bali
- Principles of Mathematical Analysis – Walter Rudin

Session – 1

Session Name: Wave Equation

Author Name: Dr. Arun Kumar Maiti

Department: Mathematics

Subject/Course: Partial Differential Equation

Course Code: CC-9

Level of students: B.Sc. Mathematics(Hons),4th Sem.

Cell Number:< 9434568316>

Session Objectives

At the end of this session, the learner will be able to:

- Derive one dimensional wave equation.
- Find D'Alembert's solution of one dimensional wave equation
- Solve various types of problems on wave equation.
- Learn about the application of wave equation in various fields, like prediction of weather and earthquakes or certain types of natural disasters.

Teaching Learning Material

- Brainstorming
- Presentation slides
- Black Board and Chalk
- Game

Session Plan

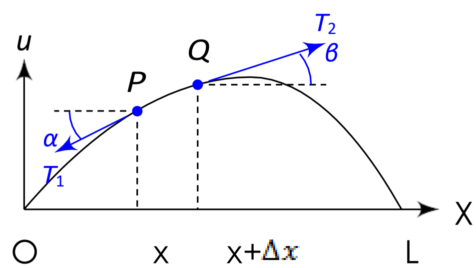
Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Review of previous lesson	Brainstorming	Question Answer	Listens Participates Discusses	Remembering Understanding
10	Derivation of one dimensional wave equation	Brainstorming	Facilitates Explains	Listens Watches Discuss	Understanding
15	D'Alembert's Solution of the wave equation	Discussions	Explains	Listens Analyzes	Analyzing Intrapersonal Logical Linguistic
10	Solution of one dimensional wave equation by method of separation of variables	Demonstration Discussions	Deduction Analyze	Listens Participates Analyzes Discusses	Remembering Understanding Interpersonal Intrapersonal Visual-spatial Logical
15	Simples problems on wave equation	Case Study Innovative conclusion	Hints for solution	Participates	Remembering Understanding Applying Knowledge to Solve Problems

Session Inputs

Idea of wave and wave equation :

The wave equation is one of the most important equations in mechanics. It describes not only the movement of strings and wires, but also the movement of fluid surfaces, e.g., water waves. These waves have many uses which are vital to our daily lives: visible light allow us to see; microwaves and radio waves allow for long-range communication via mobile phones, television and radio; infra-red waves are used in night-vision cameras and in many remote controls; and x-rays are used in medical imaging;

Derivation of one-dimensional wave equation :



We consider an elastic string which is placed along the x-axis and stretched to length L and fastened at the ends at x=0 and x=L. The string is distorted at t=0 and then released to vibrate.
The problem is to determine the string deflection u (x; t) at a point x $\in$ [0; L].
Now we consider the following assumptions:
• The tension caused by stretching the string is so large that the action of the gravitation force
can be neglected;
• The deflection happens in the vertical plane. Every particle of the string moves strictly
vertically. The deflection and the slope at every point of the string always remain small in
absolute value.

Consider the forces acting on a small portion of the string.
From Figure given above we see that there is no acceleration in the x direction. Hence the horizontal components of the tension must be constant, i.e.

$$T_1 cos\alpha = T_2 cos\beta = T \quad \text{..........................(1)}$$

By Newton's second law, in the vertical direction, we have

$$T_2 sin\beta - T_1 sin\alpha = \rho \Delta x \frac{\partial^2 u}{\partial t^2} \quad \text{.....................(2)}$$

where ρ is the mass of the undeflected string per unit length.
Dividing the last equation by (1) yields

$$\frac{T_2 sin\beta}{T_2 cos\beta} - \frac{T_1 sin\alpha}{T_1 cos\alpha} = tan\beta - tan\alpha = \frac{\rho \Delta x}{T} \frac{\partial^2 u}{\partial t^2}$$

Notice that $tan\alpha = (\frac{\partial u}{\partial x})_x$ and $tan\beta = (\frac{\partial u}{\partial x})_{x+\Delta x}$

Hence (2) is equivalent to

$$\frac{1}{\Delta x}[(\frac{\partial u}{\partial x})_{x+\Delta x} - (\frac{\partial u}{\partial x})_x] = \frac{\rho}{T} \frac{\partial^2 u}{\partial t^2}$$

Taking the limit $\Delta x \to 0$we get

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \text{ where } c^2 = \frac{T}{\rho}$$

which is the one-dimensional wave equation.

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

If I say what is the wave equation and what is the physical significance of wave equation's coefficients ?

Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting 5 responses, we will have a close look at all the points and remove irrelevant points with proper explanations. From the remaining responses the idea of continuous time and discrete time growth model can be frame as follows:

The wave equation is a partial differential equation that describes the propagation of waves such as sound waves, light waves and water waves.
The coefficients represent physical quantities such as wave speed ,density and tension.

Announcement

D'Alembert's Solution of one dimensional wave equation :

Notes

We consider the wave equation in the form $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ and introduce the transformation

u(x,t)=U(ξ,η),where ξ=x+ct and η=x−ct.

Note that ξ and η are the characteristics of the wave equation.

Now $\frac{\partial u}{\partial x} = \frac{\partial U(\xi,\eta)}{\partial x} = \frac{\partial U}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial U}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial U}{\partial \xi} + \frac{\partial U}{\partial \eta}$

Thus we have $\frac{\partial}{\partial x} \equiv \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta}$.

Similarly we can write $\frac{\partial}{\partial t} \equiv c\frac{\partial}{\partial \xi} - c\frac{\partial}{\partial \eta}$.

Thus $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \xi\,\partial \eta}$ and $\frac{\partial^2 u}{\partial t^2} = c^2(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} - \frac{\partial^2 u}{\partial \xi\,\partial \eta})$.

Therefore with the help of above transformation the above wave equation reduces to

$$\frac{\partial^2 u}{\partial \xi\,\partial \eta} = 0.$$

From which we get after integration

$$\frac{\partial u}{\partial \eta} = f(\eta)$$

A further integration gives

$$U(\xi,\eta) = \int f(\eta^/)\,d\eta^/ + F(\xi) \;= F(\xi) + G(\eta)$$

$u(x,t)$ = F(x+ct)+G(x-ct), where F and G are two arbitrary twice differentiable functions.

As t is increased we see that F(x+ct) gets horizontally shifted to the left and G(x−ct) gets horizontally shifted to the right. As a result, we conclude that the solution of the wave equation can be seen as the sum of left and right traveling waves.

Let's use initial conditions to solve for the unknown functions.
Let $u(x,0) = f(x), u_t(x,0) = g(x), |x| < \infty$.
Applying this to the general solution, we have

$$f(x) = F(x) + G(x),$$
$$g(x) = c[F'(x) - G'(x)].$$

Integrating we get

$$\frac{1}{c}\int_0^x g(s)ds = F(x) - G(x) - F(0) + G(0)$$

Adding we get

$$F(x) = \frac{1}{2}f(x) + \frac{1}{2c}\int_0^x g(s)ds + \frac{1}{2}[F(0) - G(0)]$$
$$G(x) = \frac{1}{2}f(x) - \frac{1}{2c}\int_0^x g(s)ds - \frac{1}{2}[F(0) - G(0)]$$

Now we can write out the solution

$$u(x,t) = F(x+ct) + G(x-ct)$$

Which yields the D'Alembert's Solution as

$$u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)ds$$

When $f(x)$ and $g(x)$ are defined for all $x \in \Re$,the solution is well defined.

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

If I say what do the functions f and g represent? Can D'Alembert's solution be applied to other types of waves ?

Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting few responses, we will have a close look at all the points and remove irrelevant points with proper explanations. From the remaining responses the answers of the questions can be frame as follows:

F(x+ct) represents a wave travelling to the left while G(x-ct) represents a wave travelling to the right.
With some modifications to the wave speed and boundary conditions , D'Alembert's solution can be applied to other types of waves ,like sound waves or light waves.

From the following figure it is clear that the domain of dependence of point P is red region. The point P depends on the values of u and u_t at points inside the domain. The domain of influence of P is the blue region. The points in the region are influenced by the values of u and u_t.

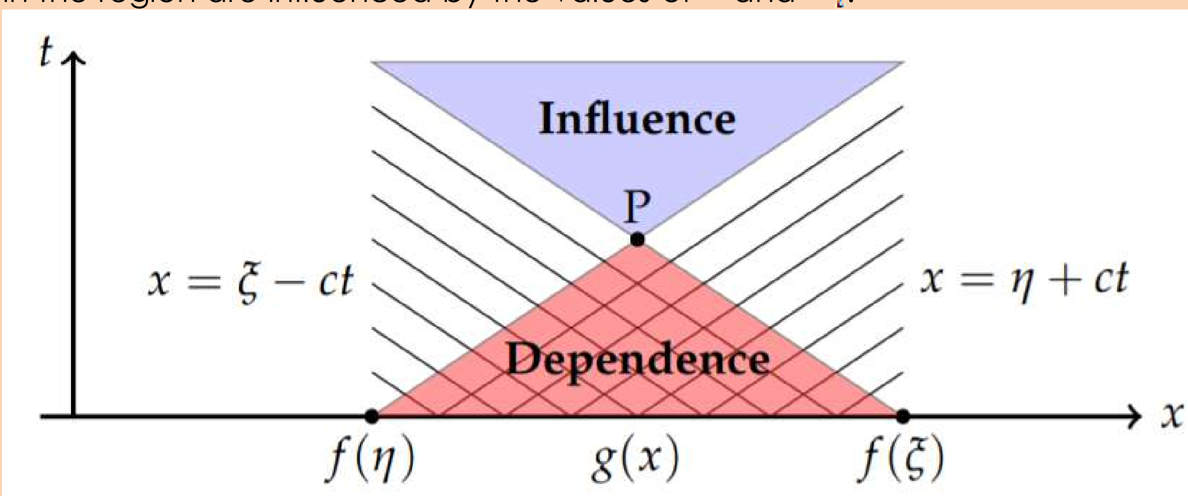


Fig: D'Alembert's Solution Of Wave Equation

Solution of one dimensional wave equation by method of separation of variables:

Consider one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary conditions $u(0,t) = u(L,t) = 0$ and the initial conditions $u(x,0)=f(x), u_t(x,0)=g(x), |x|<\infty$.

Let us put $u(x,t) = X(x)T(t)$

Then the above equation takes the form

$$\frac{X^{//}}{X} = \frac{1}{c^2}\frac{T^{//}}{T} = k, \text{say}$$

Which give $X^{//} - kX = 0$ and $T^{//} - c^2 kT = 0$

Now $u(0,t) = u(L,t) = 0$ give $X(0) = X(L) = 0$.
If $k = 0$ then $X(x) = Ax + B$. Now $X(0) = X(L) = 0$ give $A = B = 0$. Therefore $X(x) = 0$, which is not admissible. If $k > 0$ then $k = \mu^2$,say. Then $X(x) = Ce^{\mu x} + De^{-\mu x}$. Again $X(0) = X(L) = 0$ give $C + D = 0$ and $Ce^{\mu L} + De^{-\mu L}$= 0 which give $C = D = 0$ and so $X(x) = 0$. Thus $k < 0$. Let $k = -\mu^2$, then $X(x) = Ecos\mu x + Fsin\mu x$. Now $X(0) = 0$ gives $= 0$. $X(L) = 0$ gives $Fsin\mu L = 0$ $\mu = \frac{n\pi}{L}$,n is an integer.

Thus $$u(x,t) = (F_n sin\frac{n\pi}{L}x)(Gcosc\mu t + Hsinc\mu t)$$

$$= \left(G_n cos\frac{n\pi c}{L}t + H_n sin\frac{n\pi c}{L}t\right) sin\frac{n\pi}{L}x$$, where $G_n = F_n G$ and $H_n = F_n H$

$$=(G_n cos\lambda_n t + H_n sin\lambda_n t)sin\frac{n\pi}{L}x,$$ where $\lambda_n = \frac{n\pi c}{L}$

which is called the eigenfunction or characteristic function of the PDE; and λ_n's are called the eigen values of the vibrating string. The eigen functions satisfy the PDE and the boundary equation. However a single un generally does not satisfy the initial conditions. This is addressed by noting that the PDE is linear and homogeneous, hence a linear combination of the eigen functions also is a solution. Let

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t)$$

$$= \sum_{n=1}^{\infty} (G_n cos\lambda_n t + H_n sin\lambda_n t)sin\frac{n\pi}{L}x$$

we can enforce the initial condition

$$u(x,0) = \sum_{n=1}^{\infty} (G_n \ sin\frac{n\pi}{L}x) = f(x)$$

We can extend f (x) to x < 0 and x > L so that it is an odd periodic function with period 2L.(what we are interested is only the region where x 2 [0; L]). So above is in the form of a Fourier series. We thus need G_n to be the Fourier series coefficients:

$$G_n = \frac{2}{L}\int_0^L f(x)sin\frac{n\pi x}{L}dx$$

There is yet another coefficient H_n to be determined. The second

Initial condition is

$$u_t(x,0) = g(x)$$

Using the above initial condition we get

$$\sum_{n=1}^{\infty} H_n \lambda_n \ sin\frac{n\pi}{L} x = g(x)$$

From which we get

$$H_n \lambda_n = \frac{2}{L}\int_0^L g(x) \ sin\frac{n\pi x}{L} dx$$

Hence

$$H_n = \frac{2}{n\pi c}\int_0^L g(x) \ sin\frac{n\pi x}{L} dx$$

Suggested Activity:

After going through this topic, students will be able to understand how to solve one dimensional wave equation using the method of separation of variables with various initial and boundary conditions.

Simple Problem on Wave Equation:

Suppose that an infinite string has an initial displacement $u(x,0) = x^2$ and zero initial velocity. Write down the solution of the wave equation.

with simple calculations learners will be able to find the solutions of the problem as

$$u(x,t) = (x+ct)^2 - (x-ct)^2 = 4xct$$

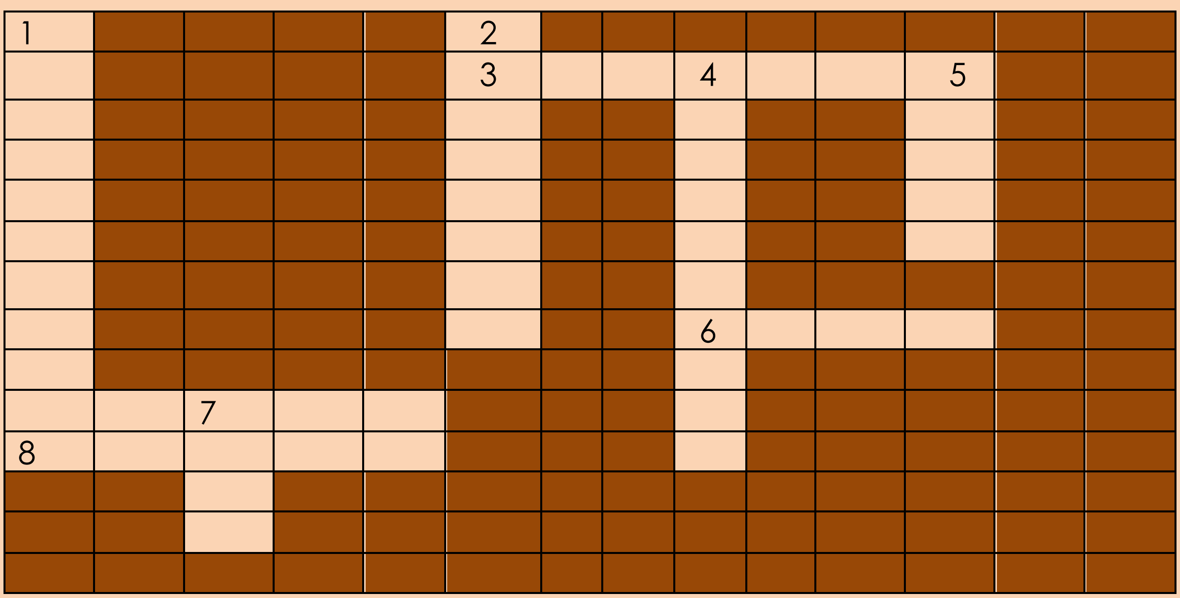
Conclusion:

The session can be concluded with a little GAME (Fastest Hand raising when you solve a cross-word puzzle)

Suggested Activity:

The following crossword puzzle will be placed in front of students to answer with a game Fastest Hand raising.

Prize: The winner with maximum answers gets a chocolate.



Clues:

Sl. No.	Across:	Sl. No.	Down
3	D'Alembert's solution of wave equation is derived on the besis of………….law.	1	In order to derive wave equation the medium is…………….
6	In D'Alembert's solution of wave equation small 'f' represents travelling wave from………………	2	D'Alembert's solution of wave equation is derived for a string of …………ends.
8	In order to derive wave equation the disturbance is……………	4	D'Alembert's solution is associate with…………….wave
		5	In wave equation the coefficient 'c' represents …….. of wave.
		7	which equation is associate with the vibration of a string?

Answer to the Crossword Puzzle:

1 H					I 2							
O					N 3	E	W	T 4	O	N	S 5	
M					F			R			P	
O					I			A			E	
G					N			V			E	
E					I			E			D	
N					T			L				
E					E			L 6	E	F	T	
O								I				
U		7 W						N				
8 S	M	A	L	L				G				
		V										
		E										

Summary

In this session, we learnt:

Derivation of Wave Equation.
D'Alembert's solution of Wave Equation.
Solution of Wave equation by using the method of separation of variables.

Assignment

Problem 1: A string is stretched and fastened to two points x=0 and x=L apart. Motion is started by displacing the string into the form from $y = f(x)$ which it is released at time t=0 . Find the displacement of any point on the string at a distance x from one end at time t.

Problem 2: A string is stretched and fastened to two points $x = 0, x = 1$ apart. Motion is started by displacing the string into the form from $y = k(1 - x)$ which it is released at time t=0 . Find the displacement of any point on the string at a distance x from one end at time t.

References

An introduction to Partial Differential Equation – Per Kristen Jakobsen.

Introduction to Partial Differential Equations with applications – E. C. Zachmanoglou, Dale W. Thoe.

Applied Partial Differential Equations with Fourier Series and Boundary Value Problems – R. Haberman.

SESSION PLAN OF CC 10

Session – 1 (2023 - 2024) (even Semester)

Session Name : Motion in Two Dimension both in Cartesian & polar Co- Ordinates

Teacher Name : Mr. Snehendu Mandal

Department : Mathematics

Subject / Course : Dynamics Of A Particle

Course Code : CC 10

Level Of Students : B.Sc. Math (Hons) , 4th Sem

Contact No :< 8436475960 >

Session Objectives :

At the end of the session students will be able to :

- *Know about the Velocity Component & Acceleration Component in Two dimension.*
- *Know about Two Dimensional Force acting on a particle .*
- *Understand about Equations of Motion in different directions of the reference frame .*
- *Find a planar curve or Path of the particle in 2D both in Cartesian & in Polar Co- ordinates .*
- *Compare between Radial & Cross Radial Component of Acceleration .*
- *Differentiate between Tangential & Normal Component of velocity & Acceleration .*
- *Solve Problems based on Two Dimensional motion .*
- *Analyze the Resulting Force , The Resulting Acceleration acting on the Particle , also find the Path of the Particle in Real life physical problem.*

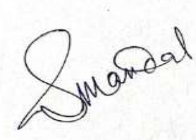
Teaching –Learning Material

- *White Board & Marker Pen*
- *Brainstorming*
- *Referrence Book*
- *Notes*

Session Plan

Time (in min)	Content	Learning aid & Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardners)
10	Review of previous lession	Brainstorming	Question Answer	Listens Participate	Remembering Understanding
07	Idea about Velocity Acceleration & Force in 2D	Brainstorming	Expains Demonstarte	Listens Discuss	Understanding
05	Formation of Equations of Motion in 2D	Case -Study	Expains & Facilitates	Analyze Understand	Analyzing Understanding Applying
08	Idea about Radial & Cross – Radial Acceleration & Velocity	Brainstorming Demonstration	Deductions & Explains	Listens Participates Discuss	Remembering Visual – spatial Intrapersonal
05	Formation Equations of Motion in polar co -- ordinates	Case -Study Group discussion	Demonstrate Analyze	Formulate Analyze	Understanding Intrapersonal
25	Various Problems on Planar motion of a particle .	Innovative Conclusion	Hints for Solution	Participates Group - Discuss	Conceptal Approach Remembering Applying knowledge to solve problems

Topic : Motion in two Dimension Teacher Name : Snehendu Mandal
College Name : Shyampur Siddheswari Mahavidyalaya , Ajodhya , Howrah



Session Inputs

Let a particle of mass m be moing in a plane under the action of any given external Force

, say F and let P (x , y) be any position of the particle at time t referred to OX , OY as Cartesian axes . If X , y be the components of the acting force parallel to OX & OY at time

t. Then by Newtons law of motion , the equations of motion of the particle parallel to OX & OY

are given by ;

$$m\ddot{x} = X$$

$$m\ddot{y} = Y$$

In Practical problems X , y may be constants or variables expressed in terms of x and y

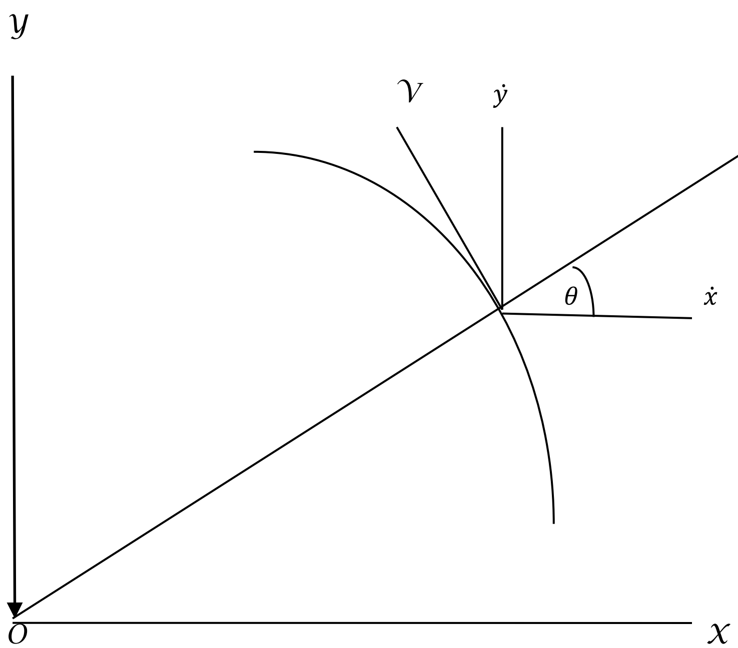
. Integrating each of the equation twice , we can find two equations involing four arbitrary

Constant , which may be determined from the given initial conditions . Finally eliminating t from the two resulting equations , we obtain a relation between x & y , which is the equation of the path traced out by the particle .

Sugessted Activity :

After Knowing & Understanding the formation of equations of motion of a particle students are able to form the equations of motion in different condition / problems .

- ### *Motion in Polar Co- Ordinates*



With reference to the rectangular axes OX & OY , Let (x , y) be the Cartesian coordinates of a moving point P at time t let (r , θ) be the polar coordinates of P at the same time t .

Then $x = r cos\theta$, $y = r sin\theta$

$$\dot{x} = \dot{r} cos\theta - \dot{\theta} r sin\theta$$

$$\dot{y} = \dot{r} sin\theta + \dot{\theta} r cos\theta$$

$$\ddot{x} = \ddot{r} cos\theta - 2\dot{r} sin\theta\theta - \dot{r} cos\theta\dot{\theta}^2 - r sin\theta\ddot{\theta}$$

$$\ddot{y} = \ddot{r} sin\theta + 2\dot{r} cos\theta\dot{\theta} - \dot{r} sin\theta\dot{\theta}^2 + r cos\theta\ddot{\theta}$$

$$
\left.
\begin{aligned}
\text{Now} \qquad u &= \dot{x}\cos\theta + \dot{y}\sin\theta = \dot{r} \\
v &= \dot{y}\cos\theta - \dot{x}\sin\theta = \dot{\theta} r \\
f_r &= \ddot{x}\cos\theta + \ddot{y}\sin\theta = \ddot{r} - r\dot{\theta}^2 \\
f_\theta &= \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})
\end{aligned}
\right\}
$$

Suggested Activity :

Students are involving to understand & formulate the different forms of velocity & accelerations in polar coordinates .

They are trying to establish th equatios of motions of a particle in polar system

Simple Problems on planar motion

Problem 1 : *A particle describes the equiangular spiral* $r = ae^{m\theta}$ *with a constant velocity . find the components of the velocity and acceleration along the Radius & Cross Radial direction .*

Problem 2 : *Find the law of force parallel to the axis of y under which a particle describes the plane curve* $xy = c^2$ *, c being a constant.*

Problem 3 *: A particle is projected with a velocity u at an angle* α *to the horizon in a medium whose resistance is* kv *. Find the time when the direction of motion will again make an angle* α *.*

Problem 4 : If the Radial & Transverse velocities of a particle are always proportional to each other , show that the path of the particle is an equiangular spiral .

Summary :

In this session we learnt ,

- *How to find the equation of motion in a planar motion both in Cartesian & in Polar coordinates .*
- *How the force acting when there is no Resistance in a medium & also the Forces in a Resisting Medium .*
- *The Law of Force may be vary as function of x or y or r or Constant force .*

Assignment :

- *Find the Law of Force in Medium whose resistance vary as k times v^n.*
- *Find the path of a plane curve traced by the particle in 2D such that its acceleration varies as v^2.*
- *Evaluate the time taken by the particle to reach the maximum height , when it is projected with a velocity u at an angle β the horizon .*
- *Calculate the Horizontal Range for a motion of a Projectile .*
- *Find the path in planar motion when the acceleration is directed towards a fixed point & always varies as the distance from it .*

References :

- *Analytical Dynamics of a particle including elents of statics – S. Ganguly , S. Saha*
- *Particle Dynamics – Dutta & Jana*
- *An elementary treatise on the dynamics of particle & Rigid bodies – S. L . Loney*
- *Textbook of Dynamics – F . Chorlton .*

Session – 4

Session Name: Graphical Representations in SageMath
Author Name: Dr. Deepshikha
Department: Mathematics
Subject/Course: Scientific computing with SageMath
Course Code: SEC-B
Unit: 1
Cell Number:< 8700527113>

Session Objectives

At the end of this session, the learner will be able to:

- Understand the commands for graphical representations of functions in SageMath.
- Learn various plotting options.
- Plot functions with asymptotes, superimposing multiple graphs in one plot like plotting a curve along with a tangent on that curve.

Teaching Learning Material

- Black Board and Chalk
- Handouts with syntaxes and examples
- Presentation slides
- Practice problems

Session Plan

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Introduction to plot commands	Brainstorming	Facilitates Explains	Listens Participates Discusses	Remembering Understanding Interpersonal Intrapersonal Verbal-linguistic
15	Formatting tools in plot commnd	Board and chalk	Facilitates Explains	Listens Participates Discusses	Understanding Remembering Analyzes Kinesthetic Interpersonal Intrapersonal Verbal-linguistic
10	Commands to plot functions with asymptotes, superimposin g multiple graphs in one plot.	Picture Presentation Board and chalk	Facilitates Explains	Listens Watches	Understanding Intrapersonal logical Verbal-linguistic
15	Problems on plot of different functions	Pictures Presentation Board and chalk	Facilitates Explains	Listens Analyzes Participates	Analyzing Understanding Interpersonal Intrapersonal Logical Linguistic
10	Conclusion, summary	Innovative conclusion (Game) Answering Crossword Puzzle	Monitors Facilitates	Participates	Remembering Understanding Applying Intrapersonal Logical

Session Inputs

Introduction to Plot Commands

Notes

In this session we learn how to plot graphs of various functions in SageMath. In the first section, we see the plotting of 2D graphs. We also learn about various formatting tools available in SageMath. Further, we study the plot of lines, circles, and functions with asymptotes. Finally, we see how to plot parametric functions, piecewise functions, functions in polar coordinates, and 3d functions.

Notes

Next, learners will understand the various command for graphical representation of various functions.

Basic definitions:

Feasible solution to a L.P.P. : A set of values of the variables, which satisfy all the constraints and all the non-negative restrictions of variables, is known as the *feasible solution* (F.S.) to the L.P.P.

Optimal solution to a L.P.P. : A feasible solution to a L.P.P. which makes the objective function an optimum is known as the *optimal solution* to the L.P.P.

Basic Feasible Solution (B.F.S.) of a L.P.P.

The solution set of a L.P.P. which is feasbile as well as basic is known as the *basic feasible solution* of the problem.

Non-degenerate B.F.S.

The solution of a L.P.P. of which all components corresponding to the basic variables are positive quantities, is known as *non-degenerate* B.F.S.

Degenerate B.F.S.

The feasible solution set of L.P.P. of which some components corresponding to the basic variables are zero, is known as the *degenerate* B.F.S.

The set of all feasible solutions to a L.P.P. $A\mathbf{x} = \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$ *is a closed convex set.*

Theorem *If a L.P.P. admits an optimal solution, then the objective function assumes the optimum value at an extreme point of the convex set generated by the set of all feasible solutions.*

Notes

How to solve LPP using Graphical method

We know from the fundamental theorem of L.P.P. (algebrical approach) that if a L.P.P. admits an optimal solution then at least one of the B.F.S. will be an optimal solution. Again we know that there is one to one correspondence between the basic feasible solutions and extreme points of the convex set of feasible solutions (In the absence of degenaracy). There are two types of convex set of feasible solutions (i) convex polyhedron, which is strictly bounded and has finite number of extreme points (ii) Convex polytope, has finite number of extreme points but not bounded from above. In the case of convex polyhedron, each and every problem has both finite maximum and finite minimum. But this is not true for a convex polytope. Here all objective functions may have finite maximum or finite minimum but not both; there is some problem which has neither finite maximum nor finite minimum and the problem has said to have unbounded solution.

Announcement

How to solve a bounded LPP

Notes

Example 1:

Solve geometrically the L.P.P.

$$\text{Maximize}, \quad z = 4x_1 + 7x_2$$

subject to

$$\begin{aligned} 2x_1 + 5x_2 &\leq 40 \\ x_1 + x_2 &\leq 11 \\ x_2 &\geq 4, \quad x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

Solution: As $x_2 \geq 4$, then the condition $x_2 \geq 0$ is redundant.

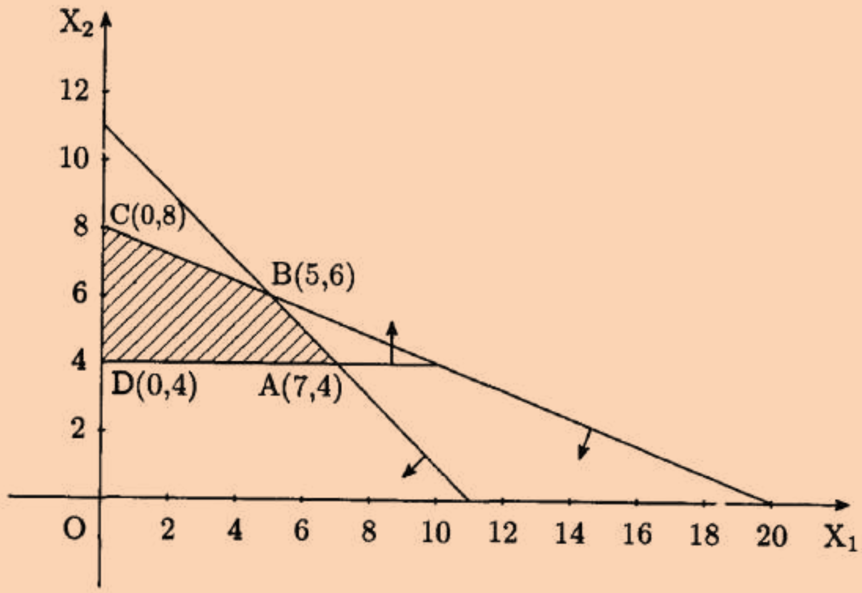
Let $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{DA}$ and $\overleftrightarrow{DC}$ denote the straight lines given by the equations $x_1 + x_2 = 11$, $2x_1 + 5x_2 = 40$, $x_2 = 4$ and $x_1 = 0$ respectively.

The convex set of feasible solutions of L.P.P. is the convex region $ABCD$. This region is also known as an admissible region. It is a strictly bounded region. The four extreme points are $A(7,4)$, $B(5,6)$, $C(0,8)$ and $D(0,4)$. The region is a convex polyhedron with finite number of extreme points.

The value of the objective function z is

$$\begin{aligned} z_1 &= 4 \times 7 + 7 \times 4 = 56 \text{ for } x_1 = 7, x_2 = 4 \text{ at } A \\ z_2 &= 4 \times 5 + 7 \times 6 = 62 \text{ for } x_1 = 5, x_2 = 6 \text{ at } B \\ z_3 &= 4 \times 0 + 7 \times 8 = 56 \text{ for } x_1 = 0, x_2 = 8 \text{ at } C \\ z_4 &= 4 \times 0 + 7 \times 4 = 28 \text{ for } x_1 = 0, x_2 = 4 \text{ at } D \end{aligned}$$

Hence the maximum value of $z = 62$ at $x_1 = 5, x_2 = 6$.



Note: (1) With the same constraints, the optimum value of the different objective functions can be determined. For example, if the objective function is $z = -2x_1 + 3x_2$ then

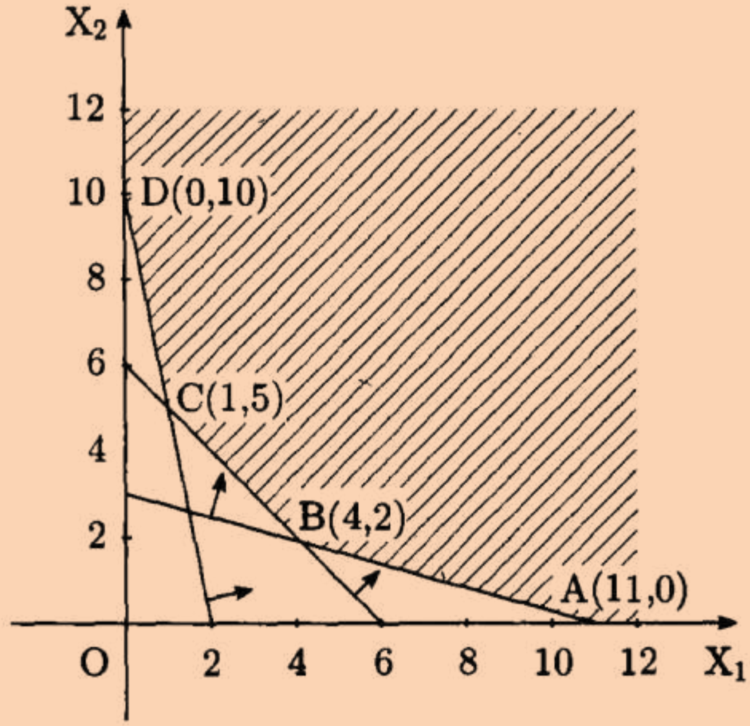
$$\begin{aligned} z_1 &= -2 \times 7 + 3 \times 4 = -2 \text{ for } x_1 = 7, x_2 = 4 \text{ at } A \\ z_2 &= -2 \times 5 + 3 \times 6 = 8 \text{ for } x_1 = 5, x_2 = 6 \text{ at } B \\ z_3 &= -2 \times 0 + 3 \times 8 = 24 \text{ for } x_1 = 0, x_2 = 8 \text{ at } C \\ z_4 &= -2 \times 0 + 3 \times 4 = 12 \text{ for } x_1 = 0, x_2 = 4 \text{ at } D \end{aligned}$$

Hence $\max z = 24$ for $x_1 = 0, x_2 = 8$ and $\min z = -2$ for $x_1 = 7, x_2 = 4$

How to solve an unbounded LPP

Example 2:

Solution: Let $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ denote respectively the straight lines given by the equations $2x_1 + 7x_2 = 22$, $x_1 + x_2 = 6$ and $5x_1 + x_2 = 10$. The admissible region is the dotted portion given in the figure. The convex set of feasible solutions is bounded from below only here, which is a convex polytope.



The four extreme points are $A(11,0), B(4,2), C(1,5)$ and $D(0,10)$. Then

$$\begin{aligned} z_1 &= 2 \times 11 + 3 \times 0 = 22 \text{ for } x_1 = 11, x_2 = 0 \text{ at } A \\ z_2 &= 2 \times 4 + 3 \times 2 = 14 \text{ for } x_1 = 4, x_2 = 2 \text{ at } B \\ z_3 &= 2 \times 1 + 3 \times 5 = 17 \text{ for } x_1 = 1, x_2 = 5 \text{ at } C \\ z_4 &= 2 \times 0 + 3 \times 10 = 30 \text{ for } x_1 = 0, x_2 = 10 \text{ at } D \end{aligned}$$

Hence $\min(z_1, z_2, z_3, z_4) = \min(22, 14, 17, 30) = 14$ for $x_1 = 4, x_2 = 2$ at B.

To verify that the minimum is finite take any two points $(4.7, 1.8)$ and $(3.9, 2.1)$ very close to the extreme point B [extreme point at which the objective function attains its lowest] on the line segments BA and BC respectively and the value of the objective function corresponding to the points $(4.7, 1.8)$ and $(3.9, 2.1)$ are $2 \times 4.7 + 3 \times 1.8 = 14.8$ and $2 \times 3.9 + 3 \times 2.1 = 14.1$ both of which are greater than the lowest value 14 at B. Thus the objective function attains its minimum value 14 at $B(x_1 = 4, x_2 = 2)$.

Note. (a) If $z \fallingdotseq 2x_1 + 3x_2$ and the problem is to be maximized then no finite value of z will be obtained. Here $\max(z_1, z_2, z_3, z_4) = 30$ at $D(0, 10)$. But for points on Dx_2, eg, at $(0, 11)$ the value of the objective function is 33 which is greater than 30 and in the same way it can be established that the objective function has no finite maximum. In that case, the problem is said to have an **unbounded solution**.

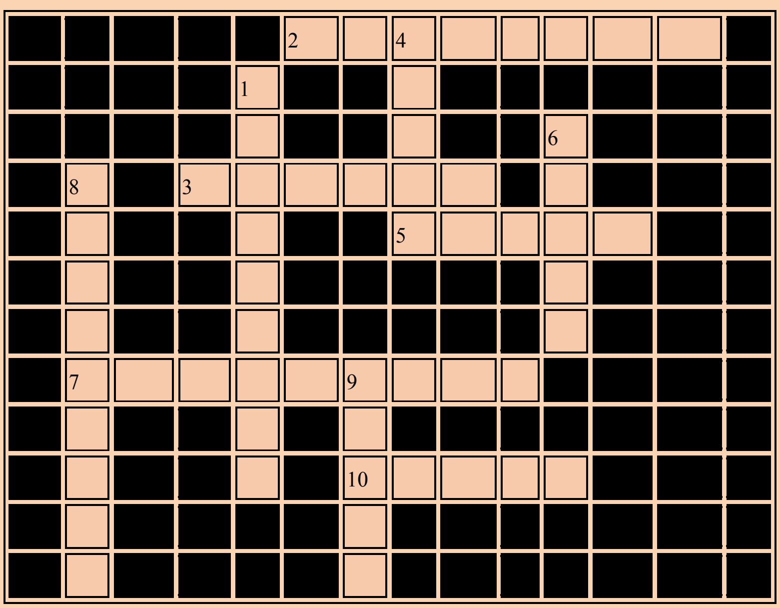
Conclusion

The session can be concluded with a little GAME (Fastest Hand raising when learners will solve a crossword puzzle).

Suggested Activity:

The following crossword puzzle will be placed in front of students to answer with a game **Fastest Hand rising.**

Crossword puzzle: **Title: LPP** Created by: Dr. Deepshikha



Across

2. The command used for ∞
4. The command for the greatest integer function
6. The option used to get gridlines on minor ticks of x-axis

Down

1. The plot option used to get thick or thin line of a plot
3. The command used to get quotient and remainder
5. The command used to round off

8. The option for getting the gridlines on a plot
10. The option used to change the ticking on the plot

numbers

7. The option used to get the dashed lines on the plot
9. The command used to name a graph of the plot

Answer to the Crossword Puzzle:

					2 I	N	4 F	I	N	I	T	Y	
				1 T			L						
				H			O			6 M			
	8 G		3 D	I	V	M	O	D		I			
	R			C			5 R	O	U	N	D		
	I			K						O			
	D			N						R			
	7 L	I	N	E	S	9 T	Y	L	E				
	I			S		I							
	N			S		10 T	I	C	K	S			
	E					L							
	S					E							

Summary

In this session, we learnt:

- Graphical method to solve LPP.
- Solution of bounded LPP using graphical method.
- Solution of unbounded LPP using graphical method.

Assignment

Solve the following LPP graphically.

1. Maximize, $z = -2x_1 + 5x_2$

$$\begin{aligned} \text{subject to } 5x_1 + 2x_2 &\leq 45 \\ 4x_1 + 5x_2 &\leq 53 \\ x_1 \quad\quad &\geq 2, \quad x_1, x_2 \geq 0. \end{aligned}$$

Find also the minimum value of z.

2. Minimize, $z = 4x_1 - 3x_2$

$$\begin{aligned} \text{subject to } 2x_1 - x_2 &\geq 4 \\ 4x_1 + 3x_2 &\leq 28, \quad x_1, x_2 \geq 0. \end{aligned}$$

3. Maximize, $z = x_1 + 3x_2$

$$\begin{aligned} \text{subject to } x_1 + x_2 &\leq 5 \\ 6x_1 - x_2 &\leq 30 \\ 9x_1 + 2x_2 &\geq 24, \quad x_1, x_2 \geq 0. \end{aligned}$$

References

- An Introduction to SAGE Programming : With Applications to SAGE, Razvan A. Mezei, Wiley.
- http://doc.sagemath.org/pdf/en/tutorial/SageTutorial.pdf
- Aniruddha Samanta and Deepshikha, Introduction to SageMath, Techno World, 2023.

SESSION PLAN OF CC 11

Session – 1 (2022-2023) (Odd Semester)

Session Name : Distribution in One Dimension

Teacher Name : Mr. Snehendu Mandal

Department : Mathematics

Subject / Course : Probability & Statistics

Course Code : CC 11

Level Of Students : B.Sc. Math (Hons) , 5^{th} Sem

Contact No : < 8436475960 >

Session Objectives :

- *Know about Random Variable*
- *Understand the Probability Distribution function*
- *Know & Analyze the Probability mass function*
- *Know about Probability density function*
- *Understand about Discrete &Continuous Distribution*
- *Know about various types of Continuous & Discrete Distribution function examples*

Teaching –Learning Material

- *Black Board & Chalk*
- *Brainstorming*
- *Referrence Book*
- *Notes*
- *Pen & Paper*

Session Plan

Time (in min)	Content	Learning aid & Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardners)
05	Review of previous lession	Brainstorming	Question Answer	Listens Participate	Remembering Understanding
10	Idea about Random Variable , Distribution function	Brainstorming	Expains Demonstarte	Listens Discuss	Understanding Analyzing
10	Concept about Discrete & Continuous Distribution	Case -Study	Expains & Facilitates	Analyze Understand	Analyzing Understanding Applying
10	Mass function & Density function	Brainstorming Demonstration	Deductions & Explains	Listens Participates Discuss	Remembering Visual - spatial Intrapersonal
10	Several types of Discrete & Continuous Distribution	Case -Study Group discussion	Demonstrate Analyze	Formulate Analyze	Understanding Intrapersonal
15	Various Problems on Probability distribution..	Innovative Conclusion	Hints for Solution	Participates Group - Discuss	Conceptal Approach Remembering Applying knowledge to solve problems

Topic :Distribution in one Dimension Teacher Name : Snehendu Mandal
College name : Shyampur Siddheswari Mahavidyalaya , Ajodhya , Howrah

Session Inputs :

Random Variable : A Random Variable X is a mapping from S to $\mathcal{R}$, such that for every real number, say x, the set $\{ w : -\infty < X(w) \leq x , w \in S \} \in \Delta$, where Δ is the class of subsets of S. The range of the mapping is called the point of spectrum of the variable X.

Now we shall write the events in a new form such as

$(-\infty < X < \leq a)$, $(X = b)$, $(-\infty < X < x)$, $(a < X \leq b)$, $(X \geq b)$ etc. etc.

Distribution Function :

Let $\mathcal{F} : \mathcal{R} \rightarrow R$ is defined by $\mathcal{F}(x) = P(-\infty < X \leq x)$

Properties of Distribution Functions :

- $0 \leq F(x) \leq 1 \quad \forall\ x \in R$
- $P(a < X \leq b) = \mathcal{F}(b) - \mathcal{F}(a)$
- $\mathcal{F}(x)$ is monotonically increasing in $(-\infty, \infty)$
- $\mathcal{F}(x)$ is continuous to the right at every point a
- $\mathcal{F}(x)$ is discontinuous to the left to every Spectrum point
- For any real constant a, $P(x = a) = \mathcal{F}(b) - \mathcal{F}(a)$
- $\mathcal{F}(\infty) = 1$ & $F(-\infty) = 0$

Probability Mass function :

Let X be a Discrete Random variable such that its Spectrum are atmost Countable Set

Let the Spectrum set be $\{ \, x_i : i = 0, \pm 1, \pm 2, \ldots \ldots \ldots \ldots \, \}$ *where*

$$\ldots \ldots \ldots \ldots < x_{-2} < x_{-1} < x_0 < x_1 \ldots \ldots \ldots \ldots \ldots$$

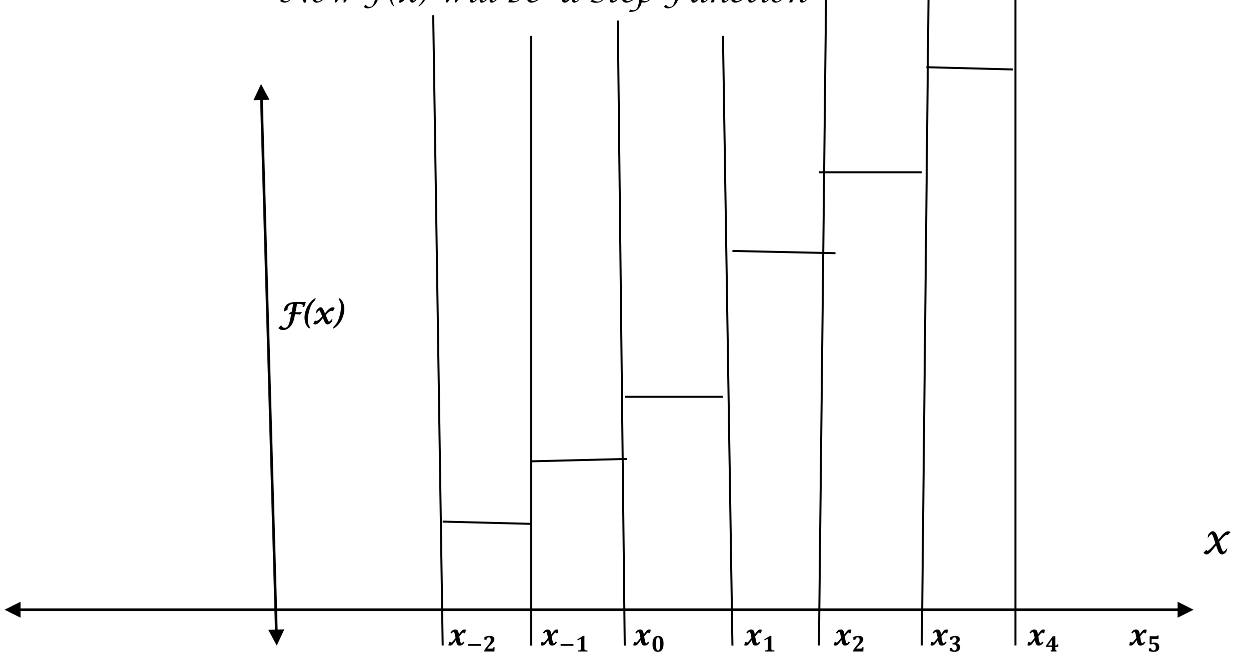
Let $P\,(X = x_i) = f_i$, x_i *being a spectrum point . A* **function** $f : R \rightarrow [0, 1]$ *is defined as follows*

$$f(x) = \begin{cases} f_i \, , \text{ if } x = x_i \text{ , which is a point of spectrum} \\ 0 \, , \text{ else where} \end{cases}$$

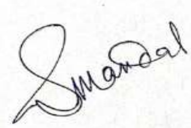
Sugesstted Activity :

$\mathcal{F}(x) = \sum P\,(X = x_j)$ *summation runs over* $-\infty$ **to** i , $x_i \le x < x_{i+1}$

Now $\mathcal{F}(x)$ *will be a Step Function*



Topic :Distribution in one Dimension Teacher Name : Snehendu Mandal
College name : Shyampur Siddheswari Mahavidyalaya , Ajodhya , Howrah



Some Important Discrete Distributions:

1) Binomial Distribution :

A discrete Random variable X having the Set { 0 , 1 , 2 , ... } as the Spectrum , is said to have Binomial distribution with parameters n , p if the P . M. F is given by

$$\left\{ \begin{aligned} f(x) &= \binom{n}{x} p^x (1-p)^x \quad , \quad \text{for } x = 0 \ , 1 , 2 , \ldots\ldots\ldots , n \\ &= 0 \quad , \text{elsewhere} \quad , \text{where } n \text{ is a positive integer} \end{aligned} \right\}$$

$0<p<1$

2) Poisson Distribution :

A Discrete random variable X having the set { 0 , 1 , 2 , ... } as the Spectrum , is said to have Poisson Distribution with parameter $\mu > 0$, if the

P.M.F is given by

$$\left\{ \begin{aligned} F(x) &= \frac{e^{-\mu}\mu^x}{x!} \quad , \ x = 0 , 1 , 2 , \ldots\ldots\ldots \\ &= 0 \quad , \text{elsewhere} \end{aligned} \right\}$$

Probability Density Function :

In case of a Continuous Random variable X , we denote F' (x) by f(x) , where f(x) is called the probability density function of x , F(x) being the

Distribution function of x . Here we assume that $f(x)$ is integrable over any bounded interal $[a, b]$, & also assume that

$\int_{-\infty}^{\infty} f(x)\,dx$ is convergent , and we write $\mathcal{F}(x) = \int_{-\infty}^{x} f(t)\,dt$

Some Examples of Continuous distributions :

1) Uniform or Rectangular Distribution

A Continuous random variable X is said to follow Uniform distribution if its P.D,F is

Given by
$$f(x) = \begin{cases} \frac{1}{b-a} & , a < x < b \\ = 0 & , \text{elsewhere} \end{cases}$$

2) Normal Distribution :

A continuous random variable X is said to have follow a Normal distribution if its P.D.F is given by

$$\mathcal{F}(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-m)^2}{2\sigma^2}} & , -\infty < x < \infty \quad , \quad \sigma > 0 \\ = 0 & , \text{elsewhere} \end{cases}$$

In particular when $m = 0$ & $\sigma = 1$, then it is called Standard Normal variate .

3) Gamma Distribution :

X is a Continuous random variable is said to follow Gamma distribution , if the p.d.f is given by

$$f(x) = \begin{cases} \dfrac{e^{-x}x^{l-1}}{\Gamma(l)} & , \quad 0 < x < \infty , \ l > 0 \\ 0 & , \quad elsewhere \end{cases}$$

Suggested activity :

Students will involve to find out or formulate some well known standard Distribution Curves & also sketch the Distribution Curves as well .

They will readily do the workout examples & solve some easy to moderate problems on regarding this session .

Some Problems on One Dimension Distribution ;

1) *Examine the following function is a P . D .F or not*

$$F(x) = \begin{cases} 2 & , \quad x = 1/2 \\ 1 & , \quad x = 1/4 \\ -1 & , \quad x = 3/4 \\ 0 & , \quad elsewhere \end{cases}$$

2) *The distribution function of random variable X is given by*

$$F(x) = \begin{cases} 0 , & x \le 0 \\ x , & 0 < x \le 1 \\ 1 , & x > 1 \end{cases}$$

Find the P . D. F &
Also find $P(.3 < X \le .5)$

3) *In the equation* $x^2 + 2x - q = 0$, q *is a random variable Uniformly distributed over the interval (0 , 2) , find the distribution function of the larger root .*

4) *Determine the value of constanyt* K *such that* $f(x)$ *defined by*

$$F(x) = \begin{cases} K\, x(1-x) , & 0 < x < 1 \\ 0 , & \text{elsewhere} \end{cases}$$

is a density function , also find the corresponding distribution function .

Summary ;

In this session we learnt ,

- *About One dimensional Random Variable ;*
- *Understand about one dimensional Distribution of Discrete & Continuous Varibale ,*
- *How to find out The Distribution Curves & Functions ,*
- *How to find out the Density curves & functions*
- *Find out Mass function of a Discrete random variable ,*
- *Understand about various types of Discrete 7 Continuous distribution functions & their corresponding mass function & Density functions ,*

Assignments :

- *Five balls are drawn from an urn containing 4 white balls , 5 balck balls . Find the probability distribution of the number of white balls drawn without replacement .*
- *Consider the random experiment of tossing a pair of coin till a head appears for the first time , Let X be the number of tosses required . Find the distribution of X .*
- *Show that the function IxI in (-1 , 1) and zero elsewhere is a possible density function , and find the corresponding distribution function .*
- *If there is war in every 15 years on average , then find the probability that there will be no war in 25 years*

References :

- *Groundwork of mathematical probability & statistics , A . Gupta*
- *Introduction to probability Models , Sheldon Ross*
- *Mathematical probability , Banerjee , De , Sen*
- *An introduction to probability theory & application , William Feller ,*
- *Introduction to Mathematical probability , Arup Mukherjee & A . Bej*

Session – 1

Session Name: Group Automorphism

Author Name: Dr. Deepshikha

Department: Mathematics

Subject/Course: Group Theory-II & Linear Algebra-II

Course Code: CC12

Unit: Group Theory-II

Cell Number:< 8700527113>

Session Objectives

At the end of this session, the learner will be able to:

- Understand the concept of isomorphism.
- Define Group Automorphism.
- Explain the various properties of automorphism.

Teaching Learning Material

- Black Board and Chalk
- Handouts with definitions and examples
- Presentation slides
- Practice problems
- Computer

Author Name: Dr. Deepshikha College Name: Shyampur Siddheswari Mahavidyalaya, Ajodhya, Howrah

Session Plan

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Introduction to automorphism	Brainstorming	Facilitates Explains	Listens Participates Discusses	Remembering Understanding Interpersonal Intrapersonal Verbal-linguistic
15	Recall groups	Board and chalk	Facilitates Explains	Listens Participates Discusses	Understanding Remembering Analyzes Kinesthetic Interpersonal Intrapersonal Verbal-linguistic
10	Definition of group isomorphism, automorphism and examples of automorphism	Picture Presentation Board and chalk	Facilitates Explains	Listens Watches	Understanding Intrapersonal logical Verbal-linguistic
15	Properties of group automorphism	Pictures Presentation Board and chalk	Facilitates Explains	Listens Analyzes	Analyzing Understanding Interpersonal Intrapersonal Logical Linguistic
10	Conclusion, summary	Innovative conclusion with a deduction of conjecture	Monitors Facilitates	Participates	Remembering Understanding Applying Intrapersonal Logical

Session Inputs

Introduction to Group Automorphism

Suppose an American and a German are asked to count a handful of objects. The American says, “One, two, three, four, five, . . . ,” whereas the German says “Eins, zwei, drei, vier, fünf, . . .” Are the two doing different things? No. They are both counting the objects, but they are using different terminology to do so. Similarly, when one person says: “Two plus three is five” and another says: “Zwei und drei ist fünf,” the two are in agreement on the concept they are describing, but they are using different terminology to describe the concept. An analogous situation often occurs with groups; the same group is described with different terminology.

Recall Groups:

The term group was used by Galois around 1830 to describe sets of one-to-one functions on finite sets that could be grouped together to form a set closed under composition. As is the case with most fundamental concepts in mathematics, the modern definition of a group that follows is the result of a long evolutionary process. Although this definition was given by both Heinrich Weber and Walter von Dyck in 1882, it did not gain universal acceptance until the 20th century.

Binary Operation: Let G be a set. A binary operation on G is a function that assigns each ordered pair of elements of G an element of G.

A binary operation on a set G, then, is simply a method (or formula) by which the members of an ordered pair from G combine to yield a new member of G. This condition is called closure. The most familiar binary operations are ordinary addition, subtraction, and multiplication of integers. Division of integers is not a binary operation on the integers because an integer divided by an integer need not be an integer.

Group: Let G be a set together with a binary operation (usually called multiplication) that assigns to each ordered pair (a, b) of elements of G an element in G denoted by ab. We say G is a group under this operation if the following three properties are satisfied.

1. **Associativity.** The operation is associative; that is, (ab)c = a(bc) for all a, b, c in G.
2. **Identity.** There is an element e (called the identity) in G such that ae = ea = a for all a in G.
3. **Inverses.** For each element a in G, there is an element b in G (called an inverse of a) such that ab = ba = e.

Announcement

Definition of Group Isomorphism:

An *isomorphism* ϕ from a group G to a group G is a one-to-one mapping (or function) from G onto $\overline{G}$ that preserves the group operation. That is,

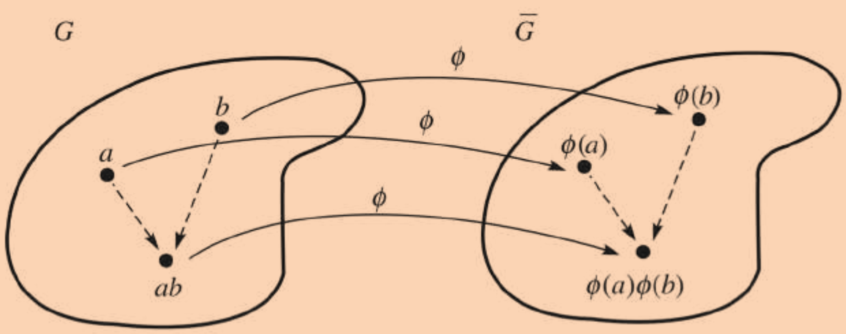
$$\phi(ab) = \phi(a)\phi(b) \qquad \text{for all } a, b \text{ in } G.$$

If there is an isomorphism from G onto $\overline{G}$, we say that G and $\overline{G}$ are *isomorphic* and write $G \approx \overline{G}$.

Notes

Suggested Activity:

This definition can be visualized as shown in the following figure. The pairs of dashed arrows represent the group operations.



It is implicit in the definition of isomorphism that isomorphic groups have the same order. It is also implicit in the definition of isomorphism that the operation on the left side of the equal sign is that of G, whereas the operation on the right side is that of $\overline{G}$. The four cases involving . and + are shown in the following Table.

G Operation	$\overline{G}$ Operation	Operation Preservation
$\cdot$	$\cdot$	$\phi(a \cdot b) = \phi(a) \cdot \phi(b)$
$\cdot$	$+$	$\phi(a \cdot b) = \phi(a) + \phi(b)$
$+$	$\cdot$	$\phi(a + b) = \phi(a) \cdot \phi(b)$
$+$	$+$	$\phi(a + b) = \phi(a) + \phi(b)$

There are four separate steps involved in proving that a group G is isomorphic to a group $\overline{G}$.

Step 1 "Mapping." Define a candidate for the isomorphism; that is, define a function ϕ from G to $\overline{G}$.

Step 2 "1–1." Prove that ϕ is one-to-one; that is, assume that $\phi(a) = \phi(b)$ and prove that $a = b$.

Step 3 "Onto." Prove that ϕ is onto; that is, for any element $\overline{g}$ in $\overline{G}$, find an element g in G such that $\phi(g) = \overline{g}$.

Step 4 "O.P." Prove that ϕ is operation-preserving; that is, show that $\phi(ab) = \phi(a)\phi(b)$ for all a and b in G.

Announcement

Definition of Group Automorphism:
An isomorphism from a group G onto itself is called an automorphism of G.

Examples of Group Automorphism

Notes

EXAMPLE The function ϕ from $\mathbf{C}$ to $\mathbf{C}$ given by $\phi(a + bi) = a - bi$ is an automorphism of the group of complex numbers under addition. The restriction of ϕ to $\mathbf{C}^*$ is also an automorphism of the group of nonzero complex numbers under multiplication.

EXAMPLE Let $\mathbf{R}^2 = \{(a, b) \mid a, b \in \mathbf{R}\}$. Then $\phi(a, b) = (b, a)$ is an automorphism of the group $\mathbf{R}^2$ under componentwise addition. Geometrically, ϕ reflects each point in the plane across the line $y = x$. More generally, any reflection across a line passing through the origin or any rotation of the plane about the origin is an automorphism of $\mathbf{R}^2$.

Suggested Activity:

Properties of Automorphisms

Aut(G) is a Group: The set of automorphisms of a group is a group under the operation of function composition.

The learners will understand the other important properties of Aut(G) with the help of following example. Then they will deduce the properties themselves.

To compute Aut(Z_{10}), we try to discover enough information about an element α of Aut(Z_{10}) to determine how α must be defined. Because Z_{10} is so simple, this is not difficult to do. To begin with, observe that once we know $\alpha(1)$, we know $\alpha(k)$ for any k, because

$$\alpha(k) = \alpha(\underbrace{1 + 1 + \cdots + 1}_{k \text{ terms}})$$
$$= \underbrace{\alpha(1) + \alpha(1) + \cdots + \alpha(1)}_{k \text{ terms}} = k\alpha(1).$$

So, we need only determine the choices for $\alpha(1)$ that make α an automorphism of Z_{10}. Since property 5 of Theorem 6.2 tells us that $|\alpha(1)| = 10$, there are four candidates for $\alpha(1)$:

$$\alpha(1) = 1; \qquad \alpha(1) = 3; \qquad \alpha(1) = 7; \qquad \alpha(1) = 9.$$

To distinguish among the four possibilities, we refine our notation by denoting the mapping that sends 1 to 1 by α_1, 1 to 3 by α_3, 1 to 7 by α_7, and 1 to 9 by α_9. So the only possibilities for Aut(Z_{10}) are α_1, α_3, α_7, and α_9.

The following Cayley tables reveal that $Aut(Z_{10})$ is isomorphic to U(10)

$U(10)$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

$Aut(Z_{10})$	α_1	α_3	α_7	α_9
α_1	α_1	α_3	α_7	α_9
α_3	α_3	α_9	α_1	α_7
α_7	α_7	α_1	α_9	α_3
α_9	α_9	α_7	α_3	α_1

Which property of $Aut(Z_n)$ can be deduced?

For every positive integer n, $Aut(Z_n)$ is isomorphic to U(n).

Conclusion

The session can be concluded with a following interesting problem by using software to form a conjecture about Aut(Dn).

Suggested Activity:

The following software computes the order of $Aut(D_n)$. Run the program for n 5 3, 5, 7, and 11. Make a conjecture about the order when n is prime. Run the program for n 5 4, 8, 16, and 32. Make a conjecture about the order when n is a power of 2. Run the program when n 5 6, 10, 14, and 22. Make a conjecture about the order when n is twice a prime. Run the program for n 5 9, 15, 21, and 33. Make a conjecture about the order when n is 3 times a prime. Try to deduce a general formula for the order of $Aut(D_n)$.

https://www.d.umn.edu/%7Ejgallian/compsciProject2018/html/chap6/ch6ex1.html

Summary

In this session, we learnt:

- Group isomorphisms.
- Definition of group automorphisms.
- Properties of group automorphisms.

Assignment

1. Prove that Aut(G) is a group?
2. Prove that Aut(Zn) is isomorphic to U(n).
3. Find Aut(Z).

References

- John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.
- Joseph A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa Publishing House, New Delhi, 1999.
- D.S. Malik, John M. Mordeson and M.K. Sen, Fundamentals of abstract algebra.
- I.N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.

Session – 1

Session Name: Population Growth Model

Author Name: Dr. Arun Kumar Maiti
Department: Mathematics

Subject/Course: Bio-Mathematics
Course Code: DSE-A
Level of students: B.Sc. Mathematics(Hons),5^{th} Sem.

Cell Number:< 9434568316>

Session Objectives

At the end of this session, the learner will be able to:

- Gain an improved understanding of mathematical models in biology.
- Derive and interpret the results obtained from models of population growth.
- Find the steady states and analyze their stabilities.
- Compare various types of population growth model.
- Solve simple problems on population growth model.

Teaching Learning Material

- Brainstorming
- Presentation slides
- Black Board and Chalk
- Game

Session Plan

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Review of previous lesson	Brainstorming	Question Answer	Listens Participates Discusses	Remembering Understanding
10	Idea of Continuous Population Growth Model	Brainstorming	Facilitates Explains	Listens Watches Discuss	Understanding
10	Model Formulation using ordinary differential equations (Malthus Model)	Case-study	Explains	Listens Analyzes	Analyzing Intrapersonal Logical Linguistic
10	Logistic Growth Model	Demonstration Discussions	Deduction Analyze	Listens Participates Analyzes Discusses	Remembering Understanding Interpersonal Intrapersonal Visual-spatial Logical
05	Limitations of Logistic Growth Model	Case Study Group Discussions	Explains	Listens Analyzes	Remembering Understanding
15	Simples problems on Population Growth Model	Case Study Innovative conclusion	Hints for solution	Participates	Remembering Understanding Applying Knowledge to Solve Problems

Session Inputs

Idea of continuous population growth model :

Notes

In an ecological system where a single species grows ,it is usually assume that the growth rate of the species is proportional to the density at that time. Mathematically it can be expressed as

$$\frac{1}{N}\frac{dN}{dt} = f(N),$$

where $f(N)$ is the proportionality constant.
Hence we begin the session with a brainstorming session.

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

If I say what is the difference between continuous time and discrete time growth model, what type of idea immediately comes to your mind.

Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting 5 responses, we will have a close look at all the points and remove irrelevant points with proper explanations. From the remaining responses the idea of continuous time and discrete time growth model can be frame as follows:

Announcement

If there is a overlap among generation then the growth of a population may be represented as continuous time. In continuous time model events can take place in every point in time and differential equations are used to formulate this type of models. On the other hand a discrete time population growth is appropriate for species which have non overlapping generation. In case of discrete time population growth difference equation is used to formulate the model.

Model formulation (Malthus Model) :

In case of Malthusian growth model per capita growth rate is independent of population size and so $f(N)$ takes the constant value r and the model equation is

$$\frac{1}{N}\frac{dN}{dt} = r$$

$$\Rightarrow N = N_0\, e^{rt},$$

Where N_0 is the initial population size.

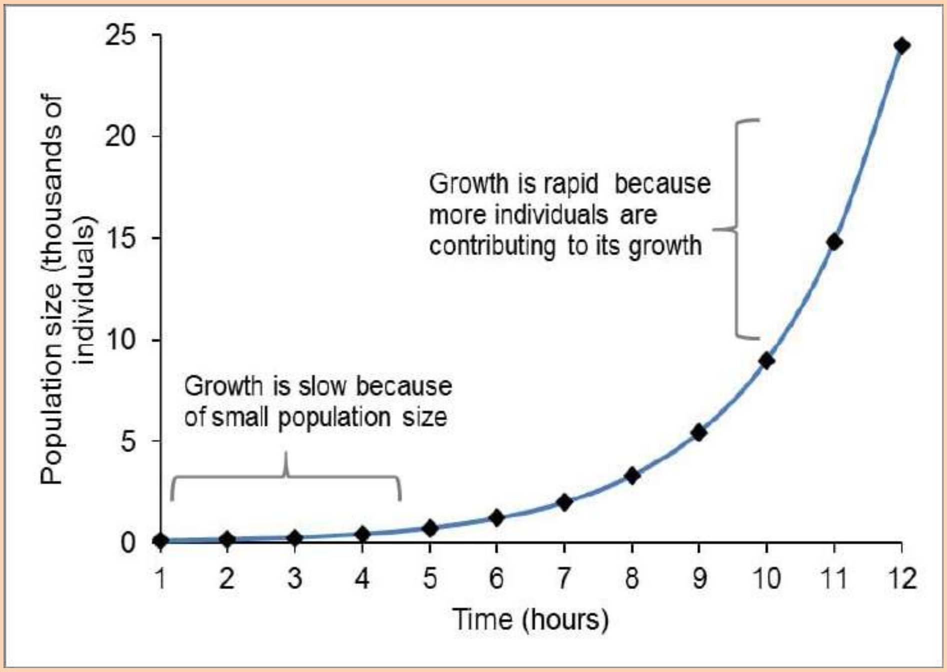
Learners will be able to find the population size after time t.

Suggested Activity:

From the following figure learners will be able to guess the population size as time t tends to infinity.

We can ask students when Malthusian model is applicable for population growth.

Malthusian Growth Pattern



Malthusian Growth Model

After looking the graph and related discussions students will understand that

a) When population size is small, then Malthusian model is appropriate.
b) Population grows infinitely as time goes to infinity.
c) Malthusian model is density independent growth model.

a) Due to limited resources (like food, light, space etc.) population can not grows exponentially.
b) Malthusian model is applicable only in the Laboratory environment.

Author Name: Dr. Arun Kumar Maiti College Name: Shyampur Siddheswari Mahavidyalaya, Ajodhya, Howrah

Logistic growth model:

Logistic growth model is density dependent. In this model $f(N)$

$$f(N) = 1 - \frac{N}{K},$$

Where K is the environmental carrying capacity. Then the model equation takes the form

$$\frac{dN}{dt} = rN(1 - \frac{N}{K})$$

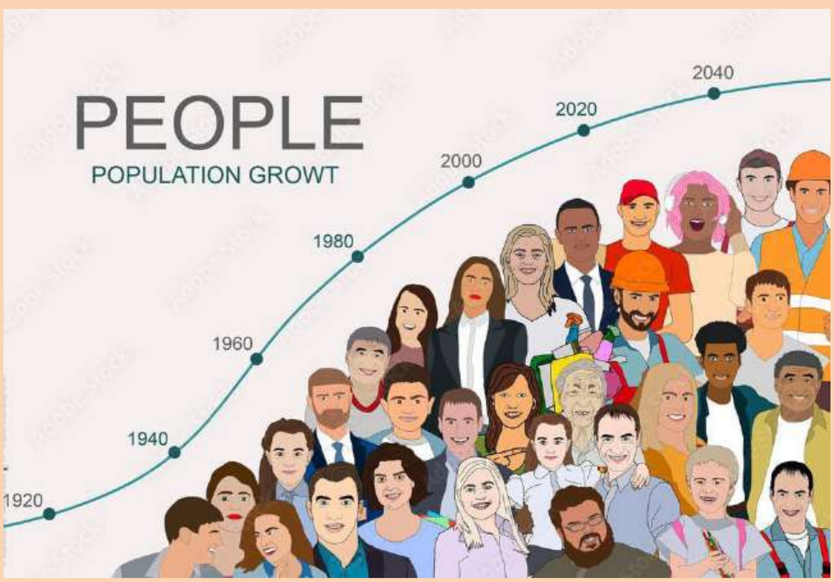
Which gives

$$N = \frac{KN_0e^{rt}}{K - N_0 + N_0e^{rt}}$$

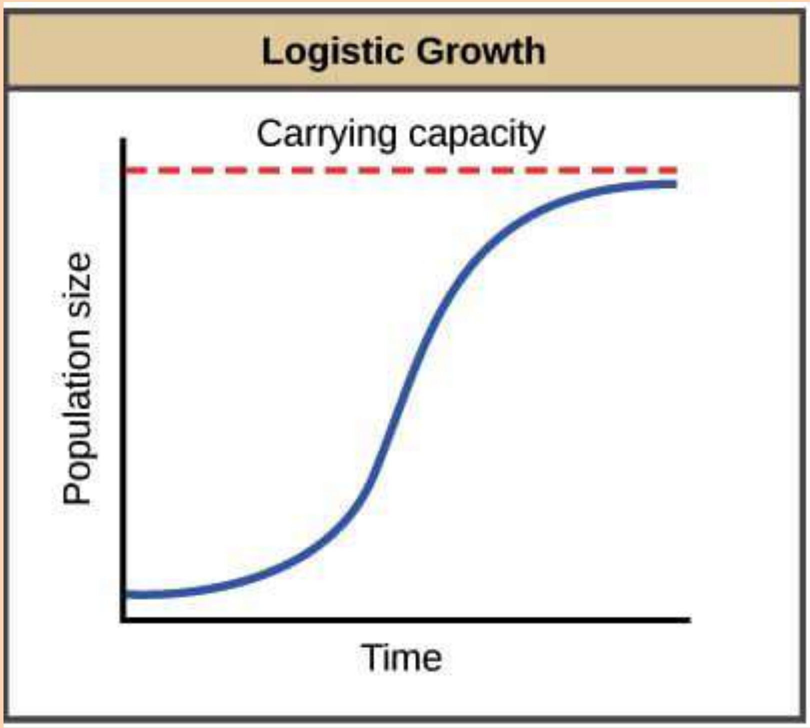
After going through this topic, students will be able to understand that population size can not exceeds the environmental carrying capacity whatever be the initial size of the population.

Suggested Activity:

With the following graphs learners will be able to identify that the growth pattern in Logistic growth model is *S*-shaped, because initially the per capita growth rate is low for small population size.



Logistic Growth Model



Limitation of Logistic Growth:

Notes

After going through discussions students' will be able to learn the follwing limitations of logistic growth model

a) Per capita growth rate is a linear function of N.
b) Population is assumed to be distributed homogeneously in their habitat.
c) The model does not consider the age structure.
d) It does not consider the effect of exogenous influences.
e) Per capita growth rate is high if the population density is very low.

Announcement

Suggested Activity:

With the above discussions learners will be able to understand that population growth follows Malthusian model when population size is low and after that it follows logistic growth pattern and ultimate reaches its carrying capacity..

Simple Problems on Population Growth:

Problem 1:

Suppose a population has 24 members at t =5 and 15 members at t=15. What was the population size at t=0 ?

Problem 2:

In an ecosystem following logistic growth model, initial population was 900 with growth rate constant value 0.1. If the carrying capacity is 1000, what is the instantaneous rate of change of population ?

with simple calculations learners will be able to find the solutions of the problems:

Problem 1: The population at time t=0 is 30.

Problem 2: The instantaneous rate of change of population is 9.

Suggested Activity:

With the above discussions learners will be able to evaluate difference between Malthusian model and logistic growth model.
By solving the above problems learners will be able to understand what type of population growth follows in nature and what are the main drawbacks of the above two models.

Logistic growth model is more appropriate than exponential growth model when population size is large.

Conclusion:

The session can be concluded with a little GAME (Fastest Hand raising when you solve a cross-word puzzle)

Suggested Activity:

The following crossword puzzle will be placed in front of students to answer with a game Fastest Hand raising.
Prize: The winner with maximum answers gets a chocolate.

1		2			3	4						
							5		6			
7												
8												

Clues:

Sl. No.	Across:	Sl. No.	Down
1	Density independent growth model is known as……model.	2	In an ecological system which type of growth is appropriate?
5	In Malthus model growth rate is………………	3	For exponential growth resource is……….
7	Exponential growth is possible when population size is………	4	The logistic growth curve is…………….
8	Logistic growth is density……..	6	In Logistic growth model ,growth is ……..

Answer to the Crossword Puzzle:

1 M	A	L 2	T	H	U 3	S 4						
		O			N	S	5 L	I	N 6	E	A	R
		G			L	H			O			
		I			I	A			N			
		S			M	P			L			
		T			I	E			I			
		I			T	D			N			
		C			E				E			
					D				A			
									R			
7 S	M	A	L	L								
8 D	E	P	E	N	D	E	N	T				

Summary

In this session, we learnt:

Models of Population growth.
Logistic Growth is more appropriate than Malthusian growth.
Exponential growth is unrealistic in nature.
Logistic growth curve is S-Shaped.
Malthus model is density independent.

Assignment

1. Find the Population doubling time in Exponential growth model.
2. Find the Population doubling time in logistic growth model.
3. From logistic growth model find the population size at time t, hence show that it reaches to its carrying capacity as t tends to infinity.
4. The nontrivial steady state of logistic growth model is
   a) Stable but not asymptotically stable b) asymptotically stable
   c)Unstable d) Both Stable and unstable .

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Mathematical Biology: I. An Introduction – J. D. Murray, Vol.17.Springer Science & Business Media.

Elements of Mathematical Ecology- M. Kot, Cambridge University Press.

Session – 4

Session Name: Graphical solution of L.P.P.

Author Name: Dr. Deepshikha

Department: Mathematics

Subject/Course: Linear Programming & Game Theory

Course Code: DSE-B(1)

Unit: 1

Cell Number:< 8700527113>

Session Objectives

At the end of this session, the learner will be able to:

- Understand the concept of bounded and unbounded LPP.
- Understand the method of solving LPP using graphical method.
- Solve Linear Programming Problems of two variables.

Teaching Learning Material

- Black Board and Chalk
- Handouts with definitions and examples
- Presentation slides
- Practice problems

Session Plan

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Introduction to basics of LPP	Brainstorming	Facilitates Explains	Listens Participates Discusses	Remembering Understanding Interpersonal Intrapersonal Verbal-linguistic
15	Steps to solve LPP using Graphical Method	Board and chalk	Facilitates Explains	Listens Participates Discusses	Understanding Remembering Analyzes Kinesthetic Interpersonal Intrapersonal Verbal-linguistic
10	Solution of bounded LPP	Picture Presentation Board and chalk	Facilitates Explains	Listens Watches	Understanding Intrapersonal logical Verbal-linguistic
15	Solution of unbounded LPP	Pictures Presentation Board and chalk	Facilitates Explains	Listens Analyzes	Analyzing Understanding Interpersonal Intrapersonal Logical Linguistic
10	Conclusion, summary	Innovative conclusion (Game) Answering Crossword Puzzle	Monitors Facilitates	Participates	Remembering Understanding Applying Intrapersonal Logical

Session Inputs

Introduction to Linear Programming Problem

Notes

With the limited available resources (such as raw materials, manpower, capital, power and technical appliance etc.) the main object of an industry is to produce different products in such a way that maximum profit may be earned by selling them at market prices. Similarly, the main aim of a housewife is to buy the goodgrains, vegetables, fruits and other food materials at a minimum cost which will satisfy the minimum need (regarding food values, calories, proteins, vitamins etc.) of the members of her family. All these (the nature of production of different commodities etc.) can be done mathematically by formulating a problem which is known as a *programming problem*. Some *restrictions* or *constraints* are to be adopted to formulate the problem. The function which is to be optimized (such as profit, cost etc. either maximized or minimized) is known as the *objective function*. Almost in all types of problems, the objective function and the constraints are of linear type, and these problems are known as the *Linear programming problems*.

Announcement

Basics of LPP:

Notes

Next, learners will recall some basic definitions regarding solution of LPP.

Basic definitions:

Feasible solution to a L.P.P. : A set of values of the variables, which satisfy all the constraints and all the non-negative restrictions of variables, is known as the *feasible solution* (F.S.) to the L.P.P.

Optimal solution to a L.P.P. : A feasible solution to a L.P.P. which makes the objective function an optimum is known as the *optimal solution* to the L.P.P.

Basic Feasible Solution (B.F.S.) of a L.P.P.

The solution set of a L.P.P. which is feasbile as well as basic is known as the *basic feasible solution* of the problem.

Non-degenerate B.F.S.

The solution of a L.P.P. of which all components corresponding to the basic variables are positive quantities, is known as *non-degenerate* B.F.S.

Degenerate B.F.S.

The feasible solution set of L.P.P. of which some components corresponding to the basic variables are zero, is known as the *degenerate* B.F.S.

The set of all feasible solutions to a L.P.P. $A\mathbf{x} = \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$ *is a closed convex set.*

Theorem *If a L.P.P. admits an optimal solution, then the objective function assumes the optimum value at an extreme point of the convex set generated by the set of all feasible solutions.*

Notes

How to solve LPP using Graphical method

We know from the fundamental theorem of L.P.P. (algebrical approach) that if a L.P.P. admits an optimal solution then at least one of the B.F.S. will be an optimal solution. Again we know that there is one to one correspondence between the basic feasible solutions and extreme points of the convex set of feasible solutions (In the absence of degenaracy). There are two types of convex set of feasible solutions (i) convex polyhedron, which is strictly bounded and has finite number of extreme points (ii) Convex polytope, has finite number of extreme points but not bounded from above. In the case of convex polyhedron, each and every problem has both finite maximum and finite minimum. But this is not true for a convex polytope. Here all objective functions may have finite maximum or finite minimum but not both; there is some problem which has neither finite maximum nor finite minimum and the problem has said to have unbounded solution.

Announcement **How to solve a bounded LPP**

Notes

Example 1:

Solve geometrically the L.P.P.

$$\text{Maximize}, \quad z = 4x_1 + 7x_2$$

subject to

$$\begin{aligned} 2x_1 + 5x_2 &\le 40 \\ x_1 + x_2 &\le 11 \\ x_2 &\ge 4, \quad x_1 \ge 0, x_2 \ge 0. \end{aligned}$$

Solution: As $x_2 \ge 4$, then the condition $x_2 \ge 0$ is redundant.

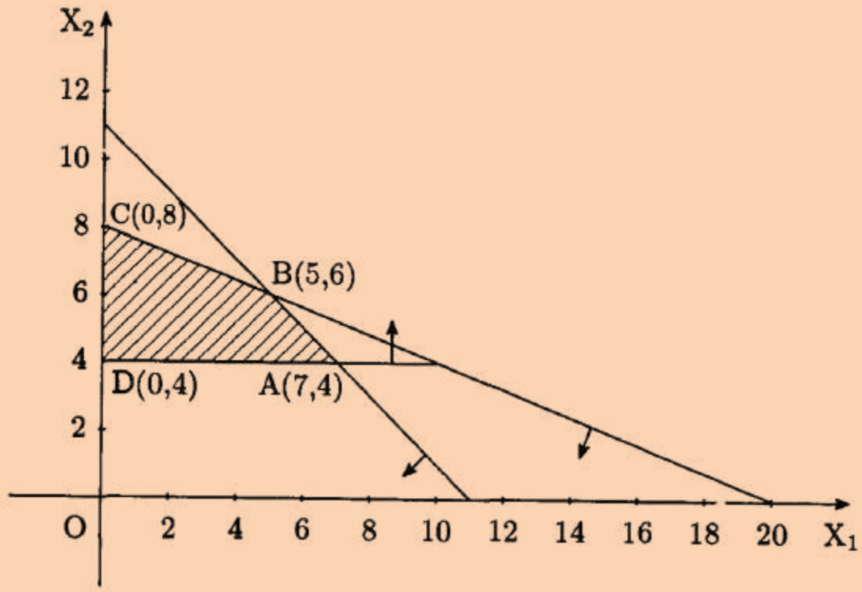
Let $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{DA}$ and $\overleftrightarrow{DC}$ denote the straight lines given by the equations $x_1 + x_2 = 11$, $2x_1 + 5x_2 = 40$, $x_2 = 4$ and $x_1 = 0$ respectively.

The convex set of feasible solutions of L.P.P. is the convex region $ABCD$. This region is also known as an admissible region. It is a strictly bounded region. The four extreme points are $A(7,4)$, $B(5,6)$, $C(0,8)$ and $D(0,4)$. The region is a convex polyhedron with finite number of extreme points.

The value of the objective function z is

$$\begin{aligned} z_1 &= 4 \times 7 + 7 \times 4 = 56 \text{ for } x_1 = 7, x_2 = 4 \text{ at } A \\ z_2 &= 4 \times 5 + 7 \times 6 = 62 \text{ for } x_1 = 5, x_2 = 6 \text{ at } B \\ z_3 &= 4 \times 0 + 7 \times 8 = 56 \text{ for } x_1 = 0, x_2 = 8 \text{ at } C \\ z_4 &= 4 \times 0 + 7 \times 4 = 28 \text{ for } x_1 = 0, x_2 = 4 \text{ at } D \end{aligned}$$

Hence the maximum value of $z = 62$ at $x_1 = 5, x_2 = 6$.



Note: (1) With the same constraints, the optimum value of the different objective functions can be determined. For example, if the objective function is $z = -2x_1 + 3x_2$ then

$$z_1 = -2 \times 7 + 3 \times 4 = -2 \text{ for } x_1 = 7, x_2 = 4 \text{ at } A$$
$$z_2 = -2 \times 5 + 3 \times 6 = 8 \text{ for } x_1 = 5, x_2 = 6 \text{ at } B$$
$$z_3 = -2 \times 0 + 3 \times 8 = 24 \text{ for } x_1 = 0, x_2 = 8 \text{ at } C$$
$$z_4 = -2 \times 0 + 3 \times 4 = 12 \text{ for } x_1 = 0, x_2 = 4 \text{ at } D$$

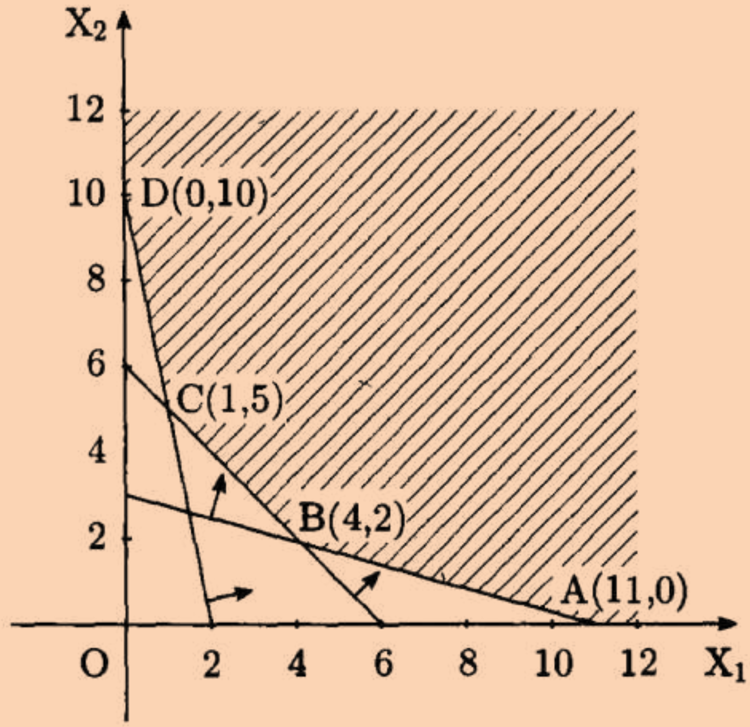
Hence $\max z = 24$ for $x_1 = 0, x_2 = 8$ and $\min z = -2$ for $x_1 = 7, x_2 = 4$

Announcement

How to solve an unbounded LPP

Example 2:

Solution: Let $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$ denote respectively the straight lines given by the equations $2x_1 + 7x_2 = 22$, $x_1 + x_2 = 6$ and $5x_1 + x_2 = 10$. The admissible region is the dotted portion given in the figure. The convex set of feasible solutions is bounded from below only here, which is a convex polytope.



The four extreme points are $A(11,0), B(4,2), C(1,5)$ and $D(0,10)$. Then

$$\begin{aligned} z_1 &= 2\times 11 + 3\times 0 = 22 \text{ for } x_1 = 11, x_2 = 0 \text{ at } A \\ z_2 &= 2\times 4 + 3\times 2 = 14 \text{ for } x_1 = 4, x_2 = 2 \text{ at } B \\ z_3 &= 2\times 1 + 3\times 5 = 17 \text{ for } x_1 = 1, x_2 = 5 \text{ at } C \\ z_4 &= 2\times 0 + 3\times 10 = 30 \text{ for } x_1 = 0, x_2 = 10 \text{ at } D \end{aligned}$$

Hence $\min(z_1, z_2, z_3, z_4) = \min(22, 14, 17, 30) = 14$ for $x_1 = 4, x_2 = 2$ at B.

To verify that the minimum is finite take any two points $(4.7, 1.8)$ and $(3.9, 2.1)$ very close to the extreme point B [extreme point at which the objective function attains its lowest] on the line segments BA and BC respectively and the value of the objective function corresponding to the points $(4.7, 1.8)$ and $(3.9, 2.1)$ are $2\times 4.7 + 3\times 1.8 = 14.8$ and $2\times 3.9 + 3\times 2.1 = 14.1$ both of which are greater than the lowest value 14 at B. Thus the objective function attains its minimum value 14 at $B(x_1 = 4, x_2 = 2)$.

Note. (a) If $z = 2x_1 + 3x_2$ and the problem is to be maximized then no finite value of z will be obtained. Here $\max(z_1, z_2, z_3, z_4) = 30$ at $D(0, 10)$. But for points on Dx_2, eg, at $(0, 11)$ the value of the objective function is 33 which is greater than 30 and in the same way it can be established that the objective function has no finite maximum. In that case, the problem is said to have an **unbounded solution**.

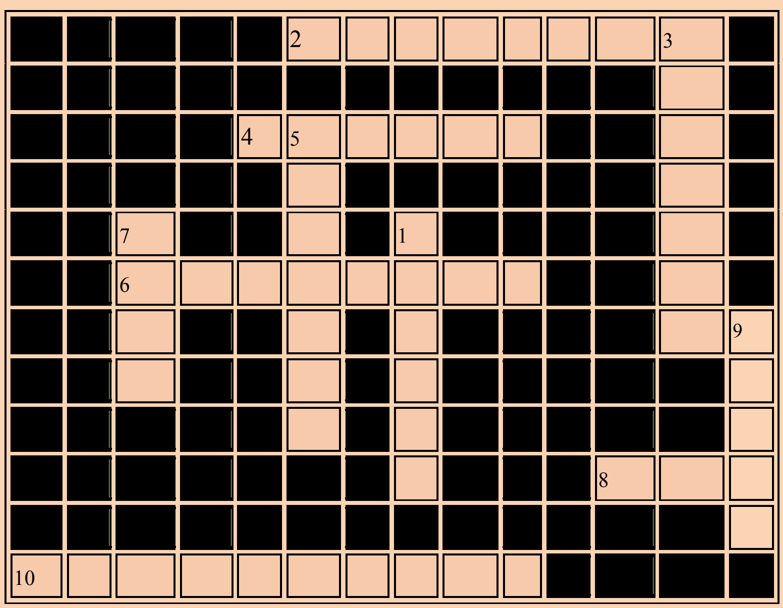
Conclusion

The session can be concluded with a little GAME (Fastest Hand raising when learners will solve a crossword puzzle).

Suggested Activity:

The following crossword puzzle will be placed in front of students to answer with a game **Fastest Hand rising.**

Crossword puzzle: **Title: LPP** Created by: Dr. Deepshikha



Across

2. The solutions which satisfy the constraints of L.P.P are known as ……… solutions
4. The set of feasible solutions is a ……… set
6. The number of extreme points of a circle is …………

Down

1. In L.P.P., L stands for …………
3. The corner points of feasible region are also known as ………… points
5. The solution which maximize or

8. If a set of m linear independent linear equations with n variable (n>m) has a F.S. then it has at least one ……..

10. The set cx=z is known as ……………..

minimize the LPP is known as ………. solution

7. The number of extreme points of a convex set generated by a pentagon is ………..

9. The polyhedron is a ………. set

Answer to the Crossword Puzzle:

					2 F	E	A	S	I	B	L	3E	
												X	
				4C	5O	N	V	E	X			T	
					P							R	
		7F			T		1L					E	
		6I	N	F	I	N	I	T	E			M	
		V			M		N					E	9C
		E			A		E						L
					L		A						O
							R				8B	F	S
													E
10H	Y	P	E	R	P	L	A	N	E				

Summary

In this session, we learnt:

- Graphical method to solve LPP.
- Solution of bounded LPP using graphical method.
- Solution of unbounded LPP using graphical method.

Assignment

Solve the following LPP graphically.

1. Maximize, $z = -2x_1 + 5x_2$

$$\begin{aligned} \text{subject to } 5x_1 + 2x_2 &\leq 45 \\ 4x_1 + 5x_2 &\leq 53 \\ x_1 &\geq 2, \quad x_1, x_2 \geq 0. \end{aligned}$$

Find also the minimum value of z.

2. Minimize, $z = 4x_1 - 3x_2$

$$\begin{aligned} \text{subject to } 2x_1 - x_2 &\geq 4 \\ 4x_1 + 3x_2 &\leq 28, \quad x_1, x_2 \geq 0. \end{aligned}$$

3. Maximize, $z = x_1 + 3x_2$

$$\begin{aligned} \text{subject to } x_1 + x_2 &\leq 5 \\ 6x_1 - x_2 &\leq 30 \\ 9x_1 + 2x_2 &\geq 24, \quad x_1, x_2 \geq 0. \end{aligned}$$

References

- Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, Linear Programming and Network Flows, 2nd Ed., John Wiley and Sons, India, 2004.
- F.S. Hillier and G.J. Lieberman, Introduction to Operations Research, 9th Ed., Tata McGraw Hill, Singapore, 2009.
- Hamdy A. Taha, Operations Research, An Introduction, 8th Ed., Prentice-Hall India, 2006.
- G. Hadley, Linear Programming, Narosa Publishing House, New Delhi, 2002.

Session – 1

Session Name: **Introduction to Metric Spaces**
Author Name: **Dr. Deepshikha**
Department: **Mathematics**
Subject/Course: **Metric Spaces & Complex Analysis**
Course Code: **CC13**
Unit: **Metric Spaces**
Cell Number:< 8700527113>

Session Objectives

At the end of this session, the learner will be able to:

- Understand the concept of metric.
- Define metric spaces.
- Explain the examples of different metrics.

Teaching Learning Material

- Black Board and Chalk
- Handouts with definitions and examples
- Presentation slides
- Practice problems

Session Plan

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Introduction to metric space	Brainstorming	Facilitates Explains	Listens Participates Discusses	Remembering Understanding Interpersonal Intrapersonal Verbal-linguistic
15	Recall distance in Euclidean spaces	Board and chalk	Facilitates Explains	Listens Participates Discusses	Understanding Remembering Analyzes Kinesthetic Interpersonal Intrapersonal Verbal-linguistic
10	Definition of metric spaces and basic properties	Picture Presentation Board and chalk	Facilitates Explains	Listens Watches	Understanding Intrapersonal logical Verbal-linguistic
15	Examples of metrics	Pictures Presentation Board and chalk	Facilitates Explains	Listens Analyzes	Analyzing Understanding Interpersonal Intrapersonal Logical Linguistic
10	Conclusion, summary	Innovative conclusion (Game) Answering Crossword Puzzle	Monitors Facilitates	Participates	Remembering Understanding Applying Intrapersonal Logical

Session Inputs

Introduction to Metric

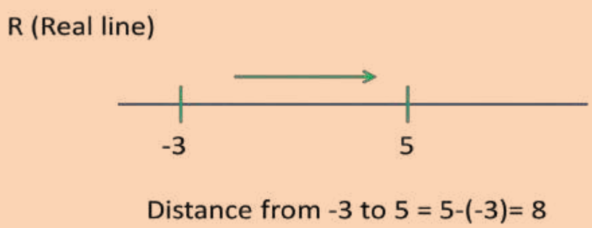
The notion of function, the concept of limit and the related concept of continuity play an important role in the study of mathematical analysis. The notion of limit can be formulated entirely in terms of distance. For example, a sequence $\{x_n\}_{n\geq 1}$ of real numbers converges to x if and only if for all $\varepsilon > 0$ there exists a positive integer n_0 such that $|x_n - x| < \varepsilon$ whenever $n \geq n_0$. A discerning reader will note that the above definition of convergence depends only on the properties of the distance $|\alpha - \beta|$ between pairs α, β of real numbers, and that the algebraic properties of real numbers have no bearing on it, except insofar as they determine properties of the distance such as,

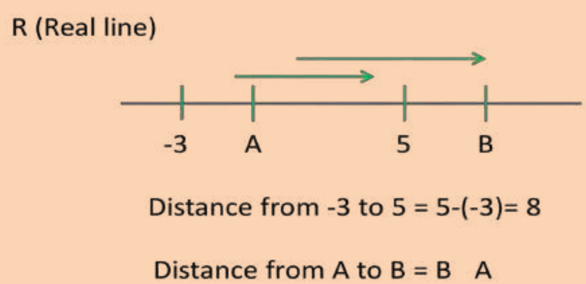
$$|\alpha - \beta| > 0 \text{ when } \alpha \neq \beta, \ |\alpha - \beta| = |\beta - \alpha| \text{ and } |\alpha - \gamma| \leq |\alpha - \beta| + |\beta - \gamma|.$$

There are many other sets of elements for which "distance between pairs of elements" can be defined, and doing so provides a general setting in which the notions of convergence and continuity can be studied. Such a setting is called a *metric space*. The approach through metric spaces illuminates many of the concepts of classical analysis and economises the intellectual effort involved in learning them.

Recall Distance Function in Euclidean Spaces:

Learners will understand how the concept of distance function in Euclidean spaces is extended to metric function through pictorial presentation.





Thus, the distance between any two points can be defined as

$$\text{distance from A to B} = \begin{cases} B - A & \text{if } B \geq A \\ A - B & \text{if } B < A \end{cases}$$

Distance using absolute value function

Distance from A to B $= |A - B|$

One can observe the following properties of the above defined distance function

Non-negativity

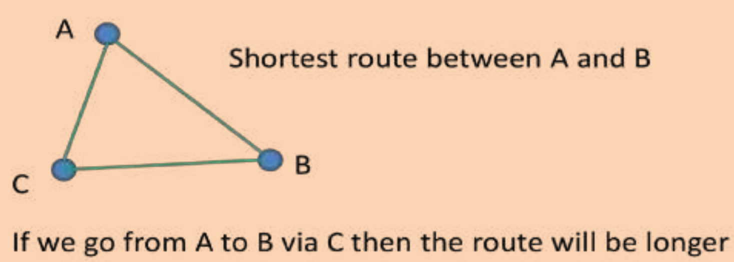
Distance from A to B ≥ 0.

Symmetry

Distance from A to B = Distance from B to A.

Distance between A and B $= 0$ if and only if A = B.

To observe the triangle inequality, see the following diagram



Distance between A & B ≤ Distance between A & C + Distance between C & B

Triangle Inequality: $|A-B| \leq |A-C|+|C-B|$

Now, we are ready to state the triangle inequality:

Triangle inequality

Distance between A and B $\leq$ Distance between A and C $+$ Distance between C and B

Definition of Metric Space:

Definition A nonempty set X with a map $d : X \times X \to \mathbf{R}$ is called a **metric space** if the map d has the following properties:

$$\begin{array}{lll} \text{(MS1)} & d(x,y) \geq 0 & x,\ y \in X; \\ \text{(MS2)} & d(x,y) = 0 & \text{if and only if } x = y; \\ \text{(MS3)} & d(x,y) = d(y,x) & x, y \in X; \\ \text{(MS4)} & d(x,y) \leq d(x,z) + d(z,y) & x, y, z \in X. \end{array}$$

The map d is called the **metric** on X or sometimes the **distance function** on X. The phrase "(X, d) is a metric space" means that d is a metric on the set X. Property (MS4) is often called the **triangle inequality.**

The four properties (MS1)–(MS4) are abstracted from the familiar properties of distance between points in physical space. It is customary to refer to elements of any metric space as **points** and $d(x, y)$ as the **distance between the points** x and y.

Next, learners will understand the different examples of metric spaces.

Euclidean or Usual Metric:

The distance function on R or R^n is a Euclidean metric. Next, learners will proved that it satisfies all the properties of metric.

Let $X = \mathbf{R}^n = \{x = (x_1, x_2, \ldots, x_n): x_i \in \mathbf{R},\ 1 \le i \le n\}$ be the set of real n-tuples. For $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ in $\mathbf{R}^n$, define

$$d(x, y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^{1/2}$$

(For $n = 2$, $d(x, y) = ((x_1 - y_1)^2 + (x_2 - y_2)^2)^{1/2}$ is the usual distance in the Cartesian plane.) To verify that d is a metric on $\mathbf{R}^n$, we need only check (MS4), i.e., if $z = (z_1, z_2, \ldots, z_n)$, we must show that $d(x, y) \le d(x, z) + d(z, y)$. For $k = 1, 2, \ldots, n$, set

$$a_k = x_k - z_k,\ b_k = z_k - y_k.$$

Then

$$d(x, z) = \left(\sum_{k=1}^{n} a_k^2 \right)^{1/2},\ d(z, y) = \left(\sum_{k=1}^{n} b_k^2 \right)^{1/2},$$

and

$$d(x, y) = \left(\sum_{k=1}^{n} (a_k + b_k)^2 \right)^{1/2}.$$

We must show that

$$\left(\sum_{k=1}^{n} (a_k + b_k)^2 \right)^{1/2} \le \left(\sum_{k=1}^{n} a_k^2 \right)^{1/2} + \left(\sum_{k=1}^{n} b_k^2 \right)^{1/2}. \qquad (*)$$

Squaring both sides of (*), and using the equality

$$(a + b)^2 = a^2 + 2ab + b^2,$$

we see that (*) is equivalent to

$$\sum_{k=1}^{n} a_k b_k \le \left(\sum_{k=1}^{n} a_k^2 \right)^{1/2} \left(\sum_{k=1}^{n} b_k^2 \right)^{1/2},$$

which is just the Cauchy-Schwarz inequality (see Theorem 1.1.4). This metric is known as the **Euclidean metric** on $\mathbf{R}^n$.

When $n > 1$, $\mathbf{R}^{n-1}$ can be regarded as a subset of $\mathbf{R}^n$ in the usual way. The metric induced on $\mathbf{R}^{n-1}$ by the Euclidean metric of $\mathbf{R}^n$ is the Euclidean metric of $\mathbf{R}^{n-1}$.

Discrete Metric:

Next, learners will see the discrete metric which is widely used in many counter examples.

Let X be any nonempty set whatsoever and let

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y. \end{cases}$$

It may be easily verified that d is a metric on X. It is called the **discrete metric** on X. If Y is a nonempty subset of X, the metric induced on Y by d is the discrete metric on Y.

Metric on C[a,b]:

The set of all continuous functions on $[a,b]$ can also be equipped with the metric

$$d(f, g) = \int_a^b |f(x) - g(x)| dx.$$

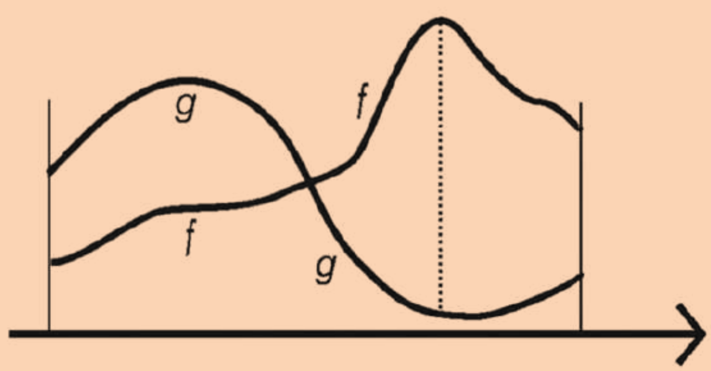


FIGURE 1.1

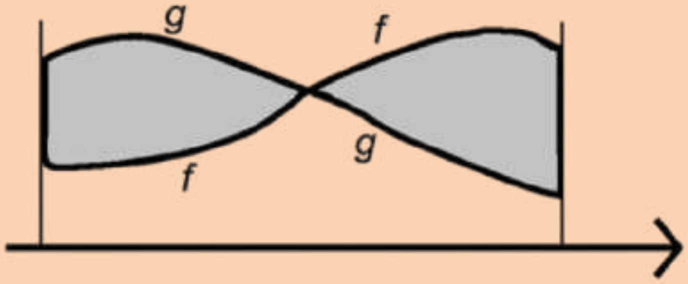


FIGURE 1.2

The measure of distance between the functions f and g represents the area between their graphs, indicated by shading in Figure 1.2. If $f, g \in C[a,b]$, then $|f - g| \in C[a,b]$, and the integral defining $d(f,g)$ is finite. It may be easily verified that d is a metric on $C[a,b]$. We note that the continuity of the functions enters into the verification of the "only if" part of (MS2).

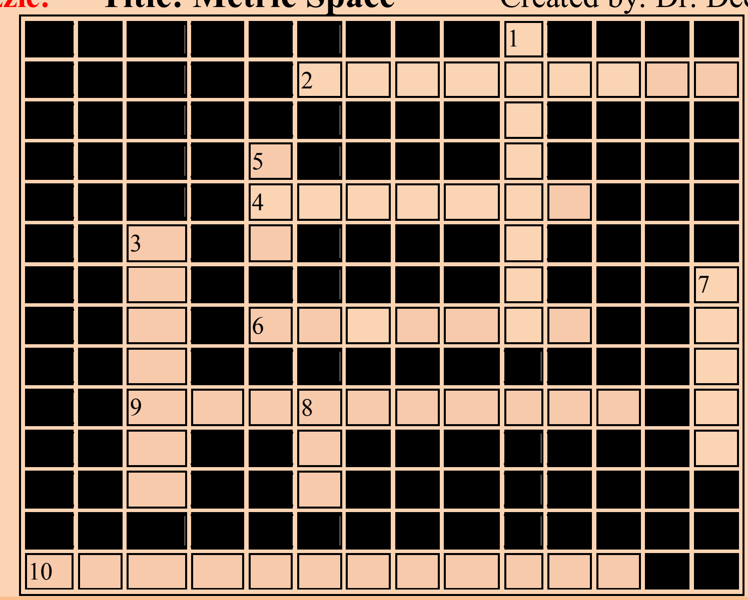
Conclusion

The session can be concluded with a little GAME (Fastest Hand raising when learners will solve a crossword puzzle).

Suggested Activity:

The following crossword puzzle will be placed in front of students to answer with a game **Fastest Hand rising.**

Crossword puzzle: **Title: Metric Space** Created by: Dr. Deepshikha



Across	Down
2. The space R^n is known as .... space	1. The metric which is either 0 or 1 is known as …………..
4. The space C^n is known as .... space	3. The metric d((a,b),(c,d))=|a-c|+|b-d| is known as ………… metric
6. B[0,1] is the space of …….. function	5. The metric d(f,g)=sup{|f(x)-g(x)|:x} is known as ……….. metric
9. C[0,1] is the space of …….. function	7. D(x,y)=|x-y| is known as ………. Metric on R^n.
10. Metric is a ………….. function	8. In taxicab metric, d((5,8),(4,7)) is ………

Answer to the Crossword Puzzle:

									1D				
					2 E	U	C	L	I	D	E	A	N
									S				
				5S					C				
				4U	N	I	T	A	R	Y			
		3T		P					E				
		A							T				7U
		X		6B	O	U	N	D	E	D			S
		I											U
		9C	O	N	8T	I	N	U	O	U	S		A
		A			W								L
		B			O								
10N	O	N	-	N	E	G	A	T	I	V	E		

Summary

In this session, we learnt:

- Properties of distance function.
- Definition of metric space.
- Different examples of metric space.

Assignment

1. For $a, b > 0$ and $0 < p < 1$, show that $(a+b)^p \le a^p + b^p$.

   Hint: The function $g(x) = \frac{(1+x)^p}{1+x^p}, x \ge 1$, has derivative $\frac{p(1+x)^{p-1}(1-x^{p-1})}{(1+x^p)^2}$, which is positive when $x > 1$. So, $g(x)$ is increasing for $x \ge 1$. Moreover, $g(x) \to 1$ as $x \to \infty$, and hence, $g(x) \le 1$ for $x \ge 1$. Put $x = a/b$ or b/a, whichever is ≥ 1.

2. Let $X = \mathbf{C}^n$ and $0 < p < 1$. Define d_p by

$$d_p(z, w) = \left(\sum_{k=1}^{n} |z_k - w_k|^p \right)^{1/p},$$

   where $z = (z_1, z_2, \ldots, z_n)$ and $w = (w_1, w_2, \ldots, w_n)$ are in $\mathbf{C}^n$. Does d_p define a metric on $\mathbf{C}^n$?
   Hint: No. For $z = (1,1,0,0,\ldots,0), \zeta = (0,1,0,0,\ldots,0)$ and $w = (0,0,0,0,\ldots,0)$, $d_p(z,w) = 2^{1/p}, d_p(z,\zeta) = 1 = d_p(\zeta,w)$, so that $d_p(z,w) > d_p(z,\zeta) + d_p(\zeta,w)$.

3. Let X be the set of all sequences $\{z_k\}_{k \ge 1}$ of numbers that are p-summable, i.e., $\sum_{k=1}^{\infty} |z_k|^p < \infty$, with $d\colon X \times X \to \mathbf{R}$ defined by

$$d(z, w) = \sum_{k=1}^{\infty} |z_k - w_k|^p, \qquad \text{where } 0 < p < 1$$

   and $z = \{z_k\}_{k \ge 1}, w = \{w_k\}_{k \ge 1}$ are in X. Then (X,d) is a metric space.
   Hint: For $z = \{z_k\}_{k \ge 1}, w = \{w_k\}_{k \ge 1}$ and $\zeta = \{\zeta_k\}_{k \ge 1}$,

$$|z_k - w_k|^p \le (|z_k - \zeta_k| + |\zeta_k - w_k|)^p \le |z_k - \zeta_k|^p + |\zeta_k - w_k|^p,$$

   using the monotonicity of the function $x \mapsto x^p$ for $x > 0$, $0 < p < 1$ and Exercise 1 above. So,

$$\sum_{k=1}^{n} |z_k - w_k|^p \le \sum_{k=1}^{n} |z_k - \zeta_k|^p + \sum_{k=1}^{n} |\zeta_k - w_k|^p \le \sum_{k=1}^{\infty} |z_k - \zeta_k|^p + \sum_{k=1}^{\infty} |\zeta_k - w_k|^p,$$

   which implies $d(z,w) \le d(z,\zeta) + d(\zeta,w)$.

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- S. Kumaresan, Topology of Metric Spaces, 2nd Ed., Narosa Publishing House, 2011.
- P. K. Jain and K. Ahmad, Metric Spaces, Narosa Publishing House.
- G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 2004..

Session – 1

Session Name: Newton Raphson Method

Author Name: Dr. Arun Kumar Maiti
Department: Mathematics

Subject/Course: Numerical Analysis
Course Code: CC-14
Level of students: B.Sc. Mathematics(Hons),6^{th} Sem.

Cell Number:< 9434568316>

Session Objectives

At the end of this session, the learner will be able to:

- Find location of the roots by tabulation method.
- Gain a deep understanding about Newton Raphson Method.
- Derive Newton Raphson formula for finding roots of an Algebraic or transcendental equations.
- Find the geometrical significance of Newton Raphson Method.
- Solve simple problems using Newton Raphson method.

Teaching Learning Material

- Brainstorming
- Presentation slides
- Black Board and Chalk
- Game

Session Plan

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Review of previous lesson	Brainstorming	Question Answer	Listens Participates Discusses	Remembering Understanding
10	Give Idea how to find Location of the roots by tabulation	Brainstorming	Facilitates Explains	Listens Watches Discuss	Understanding
10	Derivation of Newton Raphson Iteration Formula	Demonstration	Explains	Listens Analyzes	Analyzing Intrapersonal Logical Linguistic
10	Geometrical significance of Newton Raphson Method	Demonstration Discussions	Deduction Analyze	Listens Participates Analyzes Discusses	Remembering Understanding Interpersonal Intrapersonal Visual-spatial Logical
05	Condition of convergence	Group Discussions	Explains	Listens Analyzes	Remembering Understanding
15	Simples problems to find roots of non-linear equations	Innovative conclusion	Hints for solution	Participates	Remembering Understanding Applying Knowledge to Solve Problems

Session Inputs

Idea of Newton Raphson Method :

Newton Raphson Method or Newton Method is a powerful technique for solving equations numerically. It is most commonly used for approximation of the roots of the real-valued functions. Newton Rapson Method was developed by Isaac Newton and Joseph Raphson. Newton Raphson Method involves iteratively refining an initial guess to converge it toward the desired root.

Derivation of Newton - Raphson Iteration Formula:

Let x_0 be the initial approximation of the desired root and h be the small correction. then $x_0 + h$ be the exact root of the equation. Therefore

$f(x_0 + h) = 0$

$$=> f(x_0) + hf^{/}(x_0) + \frac{h^2}{2!} f^{//}(x_0) + \cdots = 0$$

As h is small, neglecting 2nd and higher power of h, we have

$f(x_0) + hf^{/}(x_0) = 0$

$$=> h = -\frac{f(x_0)}{f^{/}(x_0)}$$

Thus a better approximation of the root is

$$x_1 = x_0 - \frac{f(x_0)}{f^{/}(x_0)}$$

Repeating the above process and replacing x_0 by x_1 we get the 2nd approximation of the root is

$$x_2 = x_1 - \frac{f(x_1)}{f^{/}(x_1)}$$

Successive approximations are given by $x_3, x_4, \dots \dots \dots \dots \dots, x_{n+1}$, where

$$x_{n+1} = x_n - \frac{f(x_n)}{f^{/}(x_n)}$$

The above formula is known as Newton - Raphson iteration formula.

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

If I ask When Newton-Raphson Method fails ?, what type of idea immediately comes to your mind.

Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

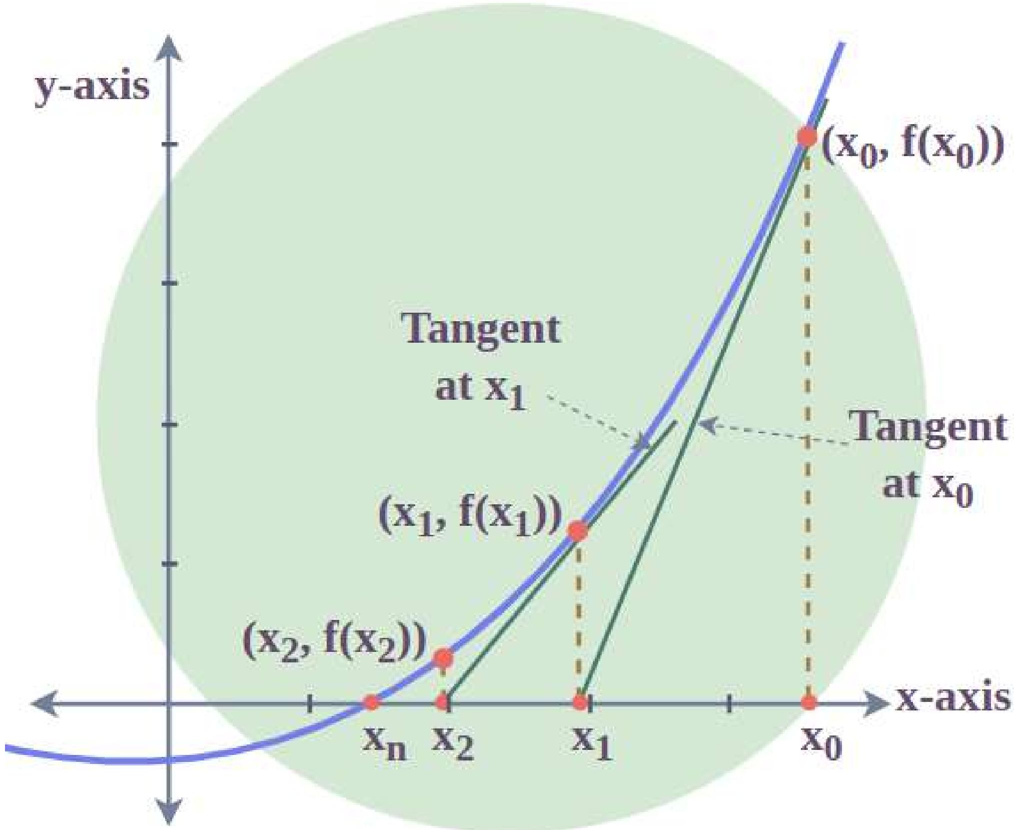
After getting 5 responses, we will have a close look at all the points and remove irrelevant points with proper explanations. From the remaining responses the correct answer can be frame as follows:

Newton-Raphson Method fails if

i) $f(x)$ is not continuously differentiable.
ii) $f'(x) = 0$ or small in the neighborhood of the root.
iii) initial approximation is not very close to the root.

Geometrical Significance:

In order to prove the validity of Newton Raphson method following steps are followed:
***Step 1*:** Draw a graph of f(x) for different values of x as shown below:



Step 2: A tangent is drawn to f(x) at x_0. This is the initial value.

Step 3: This tangent will intersect the X- axis at some fixed point (x_1,0) if the first derivative of f(x) is not zero i.e. f'(x_0) ≠ 0.

Step 4: As this method assumes iteration of roots, this x_1 is

considered to be the next approximation of the root.
Step 5: Now steps 2 to 4 are repeated until we reach the actual root x*.
Now we know that the slope-intercept equation of any line is represented as $y = mx + c$,
Where m is the slope of the line and c is the x-intercept of the line.
Using the same formula we, get

$$y = f(x_0) + f'(x_0)\ (x - x_0)$$

Here f(x_0) represents the c and f'(x_0) represents the slope of the tangent m. As this equation holds true for every value of x, it must hold true for x_1. Thus, substituting x with x_1, and equating the

equation to zero as we need to calculate the roots, we get:

$$0 = f(x_0) + f'(xx_0)\ (x_1 - x_0)$$

$$x_1 = x_0 - f(x_0)/f'(x_0)$$

Which is the Newton Raphson method formula.

Thus, Newton Raphson's method was mathematically proved and accepted to be valid.

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student.

If I ask why is the Newton-Rabson method called the method of tangent?

Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting few responses, we will have a close look at all the points and remove irrelevant points with proper explanations. The correct answer can be put as follows :

The Newton-Rabson method is called the method of tangent because it uses successive tangent to find the root of a function.

Convergence of the method :

The Newton-Raphson method tends to converge if the following condition holds true:

$$|f(x).f''(x)| < |f'(x)|^2$$

It means that the method converges when the modulus of the product of the value of the function at x and the second derivative of a function at x is lesser than the square of the modulo of the first derivative of the function at x. The Newton-Raphson Method has a convergence of order two, which means it has a quadratic convergence.

Suggested Activity:

Advantage :

Since the rate of convergence of this method is quadratic, the method converges more rapidly than Bisection Method, Regula Falsi Method.

It is to be noted that this method should not be used when the graph of the function $y = f(x)$ is nearly horizontal. The method is very useful when $f'(x)$ is large in the neighborhood of the real root, then the graph of the function $y = f(x)$ is nearly vertical and the correct value of the root can be obtained more rapidly.

Simple problems :

To make clear-cut conception teacher will provide some simple problems and solutions.

Problem 1 :

For the initial value $x_0 = 3$, approximate the root of $f(x) = x^3 + 3x + 1$.

Slution :

Given, $x_0 = 3$ and $f(x) = x^3 + 3x + 1$.

$f'(x) = 3x^2 + 3$,

f'(x_0) = 3(9) + 3 = 30.

f(x_0) = f(3) = 27 + 3(3) + 1 = 37

Using Newton Raphson method:

$x_1 = x_0 -$ f(x_0)/f'(x_0)

= 3 – 37/30
= 1.767

Problem 2 :

Find the root of the equation f($\boldsymbol{x}$) $= x^3 + 3x + 1 = \mathbf{0}$, if the initial value is 3.

Solution :

Given x_0 = 3 and f(x) $= x^3 + 3x + 1$= 0

f'(x) =$3x^2$ – 5

f'(x_0= 3) = 3 × 9 – 5 = 22

f(x_0 = 3) = 27 – 15 + 3 = 15

Using Newton Raphson method:

$x_0 = x_0$ – f(x_0)/f'(x_0)

⇒ x_1 = 3 – 15/22

⇒ x_1 = 2.3181

Using Newton Raphson method again:

x_2 = 1.9705

x_3 = 1.8504

$$x_4 = 1.8345$$

$$x_5 = 1.8342$$

Therefore, the root of the equation is approximately x = 1.834**.**

Simple Problems for home assignment :

Problem 1:

Use Newton-Rabson method to find the smallest positive of the equation

$$e^x - 3x = 0$$

correct to four decimal places.

Problem 2:

Find the cube root of 10 upto four significant figure by Newton-Rabson method.

With simple calculations learners will be able to find the solutions of the problems:

Problem 1: The root of the given equation correct to four decimal places is 1.5121 .

Problem 2: The cube root of 10 correct to four significant figures is 2.154.

Suggested Activity:

By solving the above problems learners will be able to find out the real root of algebraic and transcendental equations using Newton-Rabson method.

Newton-Rabson method converges rapidly than other methods(Bisection, Regula-Falsi, Secant)to find the root of the equation $f(x) = 0$, provided $f'(x) \neq 0$ in the neighborhood of the root.

Conclusion:

The session can be concluded with a little GAME (Fastest Hand raising when you solve a cross-word puzzle)

Suggested Activity:

The following crossword puzzle will be placed in front of students to answer with a game Fastest Hand raising.

Prize: The winner with maximum answers gets a chocolate.

	1											
							4					
					3							
2												
										8		
							7					
5			6									

Clues:

Sl. No.	Across:	Sl. No.	Down
2	If test expression is false then while loop………..	1	2nd step of for loop is checking last…………
5	While loop executes at least………	3	3rd step of for loop is…………
7	For loop is………….controlled loop.	4	Loops are used to execute a block of code…………times.
		6	Do-while loop is ……….controlled loop.
		8	The loop works as long as the input number is not………….

Answer to the Crossword Puzzle:

	E 1											
	X						M 4					
	P						U					
	R				I 3		L					
T 2	E	R	M	I	N	A	T	E	S			
	S				C		I					
	S				R		P			Z 8		
	I				E		L			E		
	O				M		E 7	N	T	R	Y	
O 5	N	C	E 6		E					O		
			X		N							
			I		T							
			T									

Summary

In this session, we learnt:

Models of Population growth.
Logistic Growth is more appropriate than Malthusian growth.
Exponential growth is unrealistic in nature.
Logistic growth curve is S-Shaped.
Malthus model is density independent.

Assignment

1. Find the Population doubling time in Exponential growth model.
2. Find the Population doubling time in logistic growth model.
3. From logistic growth model find the population size at time t, hence show that it reaches to its carrying capacity as t tends to infinity.
4. The nontrivial steady state of logistic growth model is
   a) Stable but not asymptotically stable b) asymptotically stable
   c)Unstable d) Both Stable and unstable .

References

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Mathematical Biology: I. An Introduction – J. D. Murray, Vol.17.Springer Science & Business Media.
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Session – 1

Session Name: **Simulating deterministic behavior**
Author Name: **Dr. Deepshikha**
Department: **Mathematics**
Subject/Course: **Mathematical Modelling**
Course Code: **DSE-A(2)**
Unit: **2**
Cell Number:< 8700527113>

Session Objectives

At the end of this session, the learner will be able to:

- Understand the concept of Monte Carlo simulation modelling.
- Write simulation algorithm to compute area under a curve.
- Write simulation algorithm to compute volume under a surface.

Teaching Learning Material

- Black Board and Chalk
- Handouts with definitions and examples
- Presentation slides
- Practice problems

Session Plan

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Introduction to Monte Carlo Simulation Modelling	Brainstorming	Facilitates Explains	Listens Participates Discusses	Remembering Understanding Interpersonal Intrapersonal Verbal-linguistic
15	Type of Models	Board and chalk	Facilitates Explains	Listens Participates Discusses	Understanding Remembering Analyzes Kinesthetic Interpersonal Intrapersonal Verbal-linguistic
10	Simulation of area under a curve	Picture Presentation Board and chalk	Facilitates Explains	Listens Watches	Understanding Intrapersonal logical Verbal-linguistic
15	Simulation of volume under a curve	Pictures Presentation Board and chalk	Facilitates Explains	Listens Analyzes	Analyzing Understanding Interpersonal Intrapersonal Logical Linguistic
10	Conclusion, summary	Verifying simulation algorithm using CAS	Monitors Facilitates	Participates	Remembering Understanding Applying Intrapersonal Logical

Session Inputs

Introduction to Simulation Modelling

In many situations a modeler is unable to construct an analytic (symbolic) model adequately explaining the behavior being observed because of its complexity or the intractability of the proposed explicative model. Yet if it is necessary to make predictions about the behavior, the modeler may conduct experiments (or gather data) to investigate the relationship between the dependent variable(s) and selected values of the independent variable(s) within some range. To collect the data, the modeler may observe the behavior directly. In other instances, the behavior might be duplicated (possibly in a scaled-down version) under controlled conditions.

In some circumstances, it may not be feasible either to observe the behavior directly or to conduct experiments. For instance, consider the service provided by a system of elevators during morning rush hour. After identifying an appropriate problem and defining what is meant by good service, we might suggest some alternative delivery schemes, such as assigning elevators to even and odd floors or using express elevators. Theoretically, each alternative could be tested for some period of time to determine which one provided the best service for particular arrival and destination patterns of the customers. However, such a procedure would probably be very disruptive because it would be necessary to harass the customers constantly as the required statistics were collected. Moreover, the customers would become very confused because the elevator delivery system would keep changing. Another problem concerns testing alternative schemes for controlling automobile traffic in a large city. It would be impractical to constantly change directions of the one-way streets and the distribution of traffic signals to conduct tests.

Monte Carlo Simulation Modelling:

In cases where the behavior cannot be explained analytically or data collected directly, the modeler might *simulate* the behavior indirectly in some manner and then test the various alternatives under consideration to estimate how each affects the behavior. Data can then be collected to determine which alternative is best. An example is to determine the drag force on a proposed submarine. Because it is infeasible to build a prototype, we can build a scaled model to simulate the behavior of the actual submarine. Another example of this type of simulation is using a scaled model of a jet airplane in a wind tunnel to estimate the effects of very high speeds for various designs of the aircraft. This **Monte Carlo simulation** is typically accomplished with the aid of a computer.

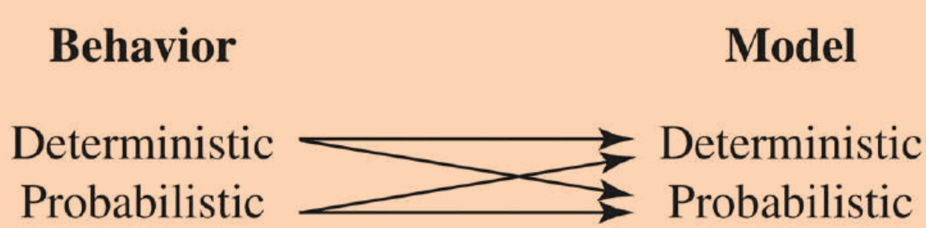
Suppose we are investigating the service provided by a system of elevators at morning rush hour. In Monte Carlo simulation, the arrival of customers at the elevators during the hour and the destination floors they select need to be replicated. That is, the distribution of arrival times and the distribution of floors desired on the simulated trial must portray a possible rush hour. Moreover, after we have simulated many trials, the daily distribution of arrivals and destinations that occur must mimic the real-world distributions in proper proportions. When we are satisfied that the behavior is adequately duplicated, we can investigate various alternative strategies for operating the elevators. Using a large number of trials, we can gather appropriate statistics, such as the average total delivery time of a customer or the length of the longest queue. These statistics can help determine the best strategy for operating the elevator system.

Notes

Next, learners will understand the types of simulation modelling.

Deterministic and Probabilistic Models:

The modeled behavior may be either deterministic or probabilistic. For instance, the area under a curve is deterministic (even though it may be impossible to find it precisely). On the other hand, the time between arrivals of customers at the elevator on a particular day is probabilistic behavior. Referring to below Figure, we see that a deterministic model can be used to approximate either a deterministic or a probabilistic behavior, and likewise, a Monte Carlo simulation can be used to approximate a deterministic or a probabilistic one. However, as we would expect, the real power of Monte Carlo simulation lies in modeling a probabilistic behavior.



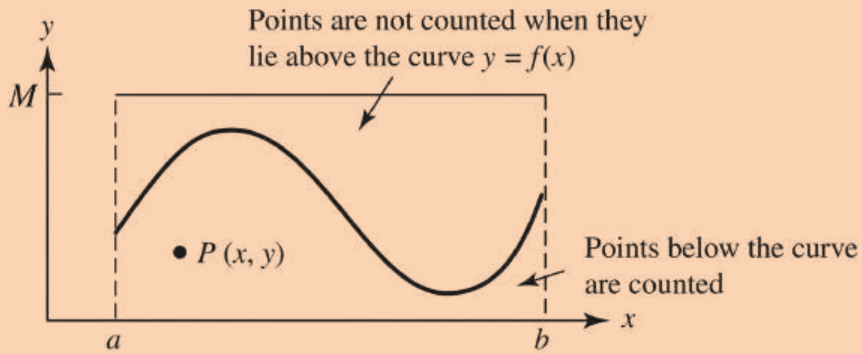
Notes

Simulating Deterministic Behavior: Area Under a Curve

Next, learners will see the how to use Monte Carlo Simulation algorithm to approximate the area under a curve.

we illustrate the use of Monte Carlo simulation to model a deterministic behavior, the area under a curve. We begin by finding an approximate value to the area under a nonnegative curve. Specifically, suppose $y = f(x)$ is some given continuous function satisfying $0 \le f(x) \le M$ over the closed interval $a \le x \le b$. Here, the number M is simply some constant that *bounds* the function. This situation is depicted in Figure Notice that the area we seek is wholly contained within the rectangular region of height M and length $b - a$ (the length of the interval over which f is defined).

Now we select a point $P(x, y)$ at random from within the rectangular region. We will do so by generating two random numbers, x and y, satisfying $a \le x \le b$ and $0 \le y \le M$, and interpreting them as a point P with coordinates x and y. Once $P(x, y)$ is selected, we ask whether it lies within the region below the curve. That is, does the y-coordinate satisfy $0 \le y \le f(x)$? If the answer is yes, then count the point P by adding 1 to some counter.



The area under the nonnegative curve $y = f(x)$ over $a \le x \le b$ is contained within the rectangle of height M and base length $b - a$.

Two counters will be necessary: one to count the total points generated and a second to count those points that lie below the curve (Figure). You can then calculate an approximate value for the area under the curve by the following formula:

$$\frac{\text{area under curve}}{\text{area of rectangle}} \approx \frac{\text{number of points counted below curve}}{\text{total number of random points}}$$

Monte Carlo Simulation Algorithm to find area under a curve

Input Total number n of random points to be generated in the simulation.

Output AREA = approximate area under the specified curve $y = f(x)$ over the given interval $a \le x \le b$, where $0 \le f(x) < M$.

Step 1 Initialize: COUNTER = 0.

Step 2 For $i = 1, 2, \dots, n$, do Steps 3–5.

Step 3 Calculate random coordinates x_i and y_i that satisfy $a \le x_i \le b$ and $0 \le y_i < M$.

Step 4 Calculate $f(x_i)$ for the random x_i coordinate.

Step 5 If $y_i \le f(x_i)$, then increment the COUNTER by 1. Otherwise, leave COUNTER as is.

Step 6 Calculate AREA = $M(b-a)$ COUNTER/n.

Step 7 OUTPUT (AREA)

STOP

Notes

Simulating Deterministic Behavior: Volume Under a Surface

Next, learners will see the how to use Monte Carlo Simulation algorithm to approximate the volume under a surface.

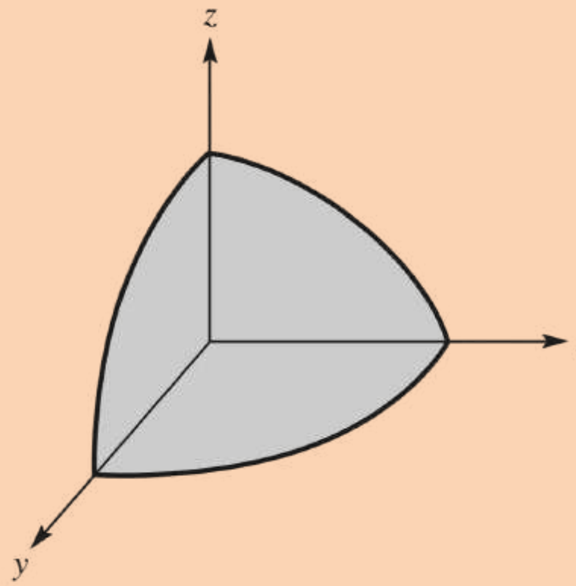
Let's consider finding part of the volume of the sphere

$$x^2 + y^2 + z^2 \le 1$$

that lies in the first octant, $x > 0$, $y > 0$, $z > 0$ (Figure.

The methodology to approximate the volume is very similar to that of finding the area under a curve. However, now we will use an approximation for the volume under the surface by the following rule:

$$\frac{\text{volume under surface}}{\text{volume of box}} \approx \frac{\text{number of points counted below surface in 1st octant}}{\text{total number of points}}$$



Volume of a sphere $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant, $x > 0$, $y > 0$, $z > 0$

Monte Carlo Simulation Algorithm to find volume under a surface

Input Total number n of random points to be generated in the simulation.

Output VOLUME = approximate volume enclosed by the specified function, $z = f(x, y)$ in the first octant, $x > 0$, $y > 0$, $z > 0$.

Step 1 Initialize: COUNTER = 0.

Step 2 For $i = 1, 2, \ldots, n$, do Steps 3–5.

Step 3 Calculate random coordinates x_i, y_i, z_i that satisfy $0 \leq x_i \leq 1$, $0 \leq y_i \leq 1$, $0 \leq z_i \leq 1$. (In general, $a \leq x_i \leq b$, $c \leq y_i \leq d$, $0 \leq z_i \leq M$.)

Step 4 Calculate $f(x_i, y_i)$ for the random coordinate (x_i, y_i).

Step 5 If random $z_i \leq f(x_i, y_i)$, then increment the COUNTER by 1. Otherwise, leave COUNTER as is.

Step 6 Calculate VOLUME $= M(d-c)(b-a)\text{COUNTER}/n$.

Step 7 OUTPUT (VOLUME)

STOP

Conclusion

The session can be concluded with verifying Monte Carlo Simulation Algorithm using SageMath.

Suggested Activity:

Use for loop command to write the program of Monte Carlo simulation algorithm to approximate the area under the curve sin(x) where x is between 0 to 1. Use different values of n and compare the results with the exact area.

Summary

In this session, we learnt:

- Simulation Modelling.
- Monte Carlo Simulation Algorithm to find the area under a curve.
- Monte Carlo Simulation Algorithm to find the volume under a surface.

Assignment

1.

Using Monte Carlo simulation, write an algorithm to calculate an approximation to π by considering the number of random points selected inside the quarter circle

$$Q : x^2 + y^2 = 1, x \geq 0, y \geq 0$$

where the quarter circle is taken to be inside the square

$$S : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1$$

Use the equation $\pi/4 = \text{area } Q/\text{area } S$.

2.

Use Monte Carlo simulation to approximate the area under the curve $f(x) = \sqrt{x}$, over the interval $\frac{1}{2} \leq x \leq \frac{3}{2}$.

Find the area trapped between the two curves $y = x^2$ and $y = 6 - x$ and the x- and y-axes.

3.

Using Monte Carlo simulation, write an algorithm to calculate that part of the volume of an ellipsoid

$$\frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{8} \leq 16$$

that lies in the first octant, $x > 0$, $y > 0$, $z > 0$.

References

- Frank R. Giordano, Maurice D. Weir and William P. Fox, A First Course in Mathematical Modeling, Thomson Learning, London and New York, 2003.
- Mathematical Modeling by Mark M. Meerschaert and Hans-Jürgen Schmidt

Session – 1

Session Name: Canonical transformation
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Subject/Course: Advanced Mechanics
Course Code: DSE-A
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Session Objectives

At the end of this session, the learner will be able to:

- Gain an improved understanding on Canonical transformation.
- Acquire knowledge about Generating function.
- Derive Canonical transformation in terms of Poisson bracket.
- Solve Various types of problems on Canonical transformation.

Teaching Learning Material

- Brainstorming
- Presentation slides
- Black Board and Chalk
- Game

Session Plan

Time (in min)	Content	Learning Aid and Methodology	Faculty Approach	Typical Student Activity	Learning Outcomes (Blooms + Gardeners)
10	Review of previous lesson	Brainstorming	Question Answer	Listens Participates Discusses	Remembering Understanding
10	Give idea about Canonical Transformation	Brainstorming	Facilitates Explains	Listens Watches Discuss	Understanding
10	GIve idea about Generating function	Demonstration Discussions	Explains	Listens Analyzes	Analyzing Intrapersonal Logical Linguistic
10	Sufficient condition of Canonical Transformation	Demonstration Discussions	Deduction Analyze	Listens Participates Analyzes Discusses	Remembering Understanding Interpersonal Intrapersonal Visual-spatial Logical
10	Define Canonical Transformation in terms of Poisson bracket	Case Study Group Discussions	Explains	Listens Analyzes	Remembering Understanding
10	Simples problems on Canonical Transformation	Case Study Innovative conclusion	Hints for solution	Participates	Remembering Understanding Applying Knowledge to Solve Problems

Session Inputs

Idea of Canonical Transformation:

Notes

In Hamiltonian mechanics, a canonical transformation is a change of canonical coordinates (q, p) → (Q, P) that preserves the form of Hamilton's equations. This is sometimes known as *form invariance*. Although Hamilton's equations are preserved, it need not preserve the explicit form of the Hamiltonian itself.

However, the class of canonical transformations is much broader, since the old generalized coordinates, momenta and even time may be combined to form the new generalized coordinates and momenta. Canonical transformations that do not include the time explicitly are called restricted canonical transformations (many textbooks consider only this type).

In the Hamiltonian approach, we're in *phase* space with a coordinate system having positions and momenta on an equal footing. It is therefore possible to think of more general transformations than the point transformation (which was restricted to the position coordinates).

We can have transformations that mix up position and momentum variables:

$$Q_i = Q_i(p_i, q_i, t)$$

$$P_i = P_i(p_i, q_i, t)$$

where (p_i, q_i) means the whole set of the original variables. In those original variables, the equations of motion had the nice *canonical* Hamilton form,

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Things won't usually be that simple in the new variables, but it does turn out that many of the "natural" transformations that arise in dynamics, such as that corresponding to going forward in time, *do* preserve the form of Hamilton's canonical equations, that is to say

$$\dot{Q}_i = \partial \bar{H} / \partial P_i\ , \dot{P}_i = -\ \partial \bar{H} / \partial Q_i,$$ for the new $\bar{H}\ (P, Q)$.

A transformation that retains the canonical form of Hamilton's equations is said to be **canonical**.

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

If I ask Is point transformation canonical ? Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting 2 responses, we will have a close look at all the points and remove irrelevant points with proper explanations. From the remaining responses the correct answer can be frame as follows:

Since Lagrangian mechanics is based on generalized coordinates, transformations of the coordinates **q** → **Q** do not affect the form of Lagrange's equations and, hence, do not affect the form of Hamilton's equations if the momentum simultaneously changed by a Legendre transformation into

$$P_i = \frac{\partial L}{\partial \dot{Q}_i}$$

where (Q_i, P_i) are the new co-ordinates and new momenta respectively.

Therefore, coordinate transformations (also called *point* transformations) are a type of canonical transformation.

Generating Function :

From Hamilton's Principle we have for old coordinate system (p_j, q_j)

$$\delta \int_{t_1}^{t_2} [\sum p_j \dot{q}_j - H(q_j, p_j, t)]\, dt = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

In terms of new coordinate system (Q_j, P_j)

$$\delta \int_{t_1}^{t_2} [\sum P_j \dot{Q}_j - \bar{H}(Q_j, P_j, t)] dt = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2) we can write

$$\delta \int_{t_1}^{t_2} [\sum p_j \dot{q}_j - H(q_j, p_j, t)) - (\sum P_j \dot{Q}_j - \bar{H}(Q_j, P_j, t))]\, dt = 0 \quad \ldots\ldots\ldots(3)$$

Equation (3) will not be affected if we add to or subtract from it a total time derivative of a function $F = F(q, p, t)$, because

$$\delta \int_{t_1}^{t_2} \frac{dF}{dt} dt = \delta [F(q, p, t)]_{t_1}^{t_2} = [\frac{\partial F}{\partial q_j} \delta q_j]_{t_1}^{t_2} + [\frac{\partial F}{\partial p_j} \delta p_j]_{t_1}^{t_2} = 0,$$

since at end points the variation in q_j and p_j vanish.

Therefore we can write equation (3) as

$$\delta \int_{t_1}^{t_2}[\sum p_j \dot{q}_j - H\,(q_j\,,p_j,\mathrm{t})) - (\sum P_j \dot{Q}_j - \bar{H}\,(Q_j\,,P_j,\mathrm{t}))] \;-\; \frac{dF}{dt}]\mathrm{dt} = 0$$

Thus it follows that

$$\left(\sum p_j \dot{q}_j - H\right) - \left(\sum P_j\,\dot{Q}_j - \bar{H}\right) = \frac{dF}{dt}$$

Now F is a function of both old and new set of co-ordinates and therefore out of 2n variables, n should be taken from new and n from old set that is, one variable should be out of q_j, p_j and other should be from Q_j, P_j set. Thus following four forms of

function F are possible

$F_1\,(\mathrm{q, Q, t}), F_2\,(\mathrm{q, P, t}), F_3\,(\mathrm{p, Q, t}), F_4\,(\mathrm{p, P, t})$

The function F is called the generating function of the transformation.

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

If I ask what will be the relation between q_j, p_j and Q_j, P_j in terms of $F_1\,(\mathrm{q, Q, t})$? Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting few responses we will have a close look at all the points and remove irrelevant points with proper explanations. From the remaining responses the correct answer can be frame as follows:

Announcement

We have

$$\left(\sum p_j \dot{q}_j - H\right) - \left(\sum P_j \dot{Q}_j - \bar{H}\right) = \frac{dF}{dt}$$

$$\left(\sum p_j \dot{q}_j - H\right) - \left(\sum P_j \dot{Q}_j - \bar{H}\right) = \frac{\partial F_1}{\partial t} + \sum \left(\frac{\partial F_1}{\partial q_j}\dot{q}_j + \frac{\partial F_1}{\partial Q j}\dot{Q}_j\right)$$

Since q_j and Q_j are treated as independent, we have

$$p_j = \frac{\partial F_1}{\partial q_j}, P_j = \frac{\partial F_1}{\partial Q_j}, \bar{H} = H + \frac{\partial F_1}{\partial t}$$

Which are the required relationship.

Condition for a transformation to be canonical:

Notes

If the expression

$$\sum_j P_j dQ_j - p_j dq_j)$$

Or,

$$\sum_j (p_j dq_j - P_j dQ_j)$$

be an exact differential , then the transformation from (q_j, p_j) set to (Q_j, P_j) is canonical.

Proof: We know that for a transformation to be canonical if

$$\left(\sum_{j} p_j \dot{q}_j - H\right) - \left(\sum_{j} P_j \dot{Q}_j - \bar{H}\right) = \frac{dF}{dt}$$

Suppose generating function F does not include time explicity then

$$\bar{H} = H + \frac{\partial F}{\partial t} = H$$

So the above equation becomes

$$\sum p_j \dot{q}_j - \sum P_j \dot{Q}_j = \frac{\partial F}{\partial t}$$
$$\sum_{j} (p_j dq_j - P_j dQ_j) = dF$$

Where dF is exact differential of F

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

What are the advantages of canonical transformation? Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting few responses we will have a close look at all the points and remove irrelevant points with proper explanations. From the remaining responses the correct answer can be frame as follows:

1) Canonical transformation can simplify complex systems by transforming them into more manageable forms.

2) Canonical transformation can separate variables , making it easier to solve problems.

3) Canonical transformation can identify constants of motion, which are essential in understanding the behavior of physical system.

4) Canonical transformation can introduce action-angle variables, which are useful in understanding periodic motion.

Canonical transformation in terms of Poisson bracket :

We have shown that a canonical transformation can be generated from functions $F_1(q_j,Q_j,t), F_2(q_j,P_j,t), F_3(p_j,Q_j,t), F_4(p_j,P_j,t)$

In case of generating function F_1 we have

$$p_j = \frac{\partial F_1}{\partial q_j}, P_j = -\frac{\partial F_1}{\partial Q_j}$$

From these relations it is quite easy to see that

$$\frac{\partial p_j}{\partial Q_i} = \frac{\partial^2 F_1}{\partial Q_i \partial q_j} = -\frac{\partial P_i}{\partial q_j}$$

Similarly in the case of $F_2(q_j,P_j,t)$

$$p_j = \frac{\partial F_2}{\partial q_j}, Q_j = \frac{\partial F_2}{\partial P_j}$$

Thus

$$\frac{\partial p_j}{\partial P_i} = \frac{\partial^2 F_2}{\partial P_i \partial q_j} = \frac{\partial Q_i}{\partial q_j}$$

Similarly from F_3 and F_4 we can find that

$$\frac{\partial q_j}{\partial Q_i} = \frac{\partial P_i}{\partial p_j}$$

$$\frac{\partial q_j}{\partial P_i} = -\frac{\partial Q_i}{\partial p_j}$$

Now we write the fundamental Poisson bracket as

$$[Q_i,P_j]_{q,p} = \sum_k \left(\frac{\partial Q_i}{\partial q_k}\frac{\partial P_j}{\partial p_k} - \frac{\partial Q_i}{\partial p_k}\frac{\partial P_j}{\partial q_k}\right)$$

$$= \sum_k \left(\frac{\partial Q_i}{\partial q_k}\frac{\partial q_k}{\partial Q_j} + \frac{\partial Q_i}{\partial p_k}\frac{\partial p_k}{\partial Q_j}\right)$$

$$= \frac{\partial Q_i}{\partial Q_j} = \delta_{ij} = [Q_i,P_j]_{Q,P}$$

Similarly

$$[Q_i, Q_j]_{q,p} = 0 = [Q_i, P_j]_{Q,P}$$
$$[P_i, P_j]_{q,p} = 0 = [P_i, P_j]_{Q,P}$$

Suggested Activity:

We can conduct brainstorming activity by posing following question to the student

If I ask one question what will be the value of $[Q_i, P_i]_{q,p}$? Learners can be given one minute time to respond. They can be chosen arbitrarily and will be asked to share whatever comes in their mind. After getting individual responses, the facilitator will enlist them on the board.

After getting few responses we will have a close look at all the points and write down the correct value of $[Q_i, P_i]_{q,p}$=1, since the value of Kronecker delta δ_{ij} is 1 if $i = j$ and the value will be zero if $i \neq j$.

Simple Problems on Canonical transformation:

Problem 1:

Show that the transformation $q = \sqrt{2P}\, sinQ$

$$p = \sqrt{2P}\, cosQ$$

is canonical.

Problem 2:

Show that the transformation $P = qcotp$

$$Q = \log\left(\frac{sinp}{q}\right)$$

is canonical.

with simple calculations learners will be able to find the solutions of the problems:

Problem 1:
We have

$$P = \frac{1}{2}(p^2 + q^2) \quad Q = tan^{-1}(q/p)$$

Now

$$pdq - PdQ = pdq - \frac{1}{2}(p^2 + q^2)\frac{1}{1 + \frac{q^2}{p^2}}\frac{pdq - qdp}{p^2}$$

$$= pdq - \frac{1}{2}(pdq - qdp) = \frac{1}{2}d(pq) = d(\frac{1}{2}pq)$$

Therefore the transformation is canonical.

Problem 2:
Now

$$pdq - PdQ = pdq - (qcotp)\frac{q}{sinp}\frac{cospdp.q - sinp.dq}{q^2}$$

$$= pdq - (qcotp)\left(cotpdp - \frac{dq}{q}\right)$$

$$= pdq - qcot^2pdp + cotpdq$$

$$= pdq - q(cosec^2p - 1)dp + cotp.dq$$

$$= d(q(p + cotp))$$

Therefore the given transformation is canonical.

Suggested Activity:

With the above discussions learners will be able to check whether a transformation is canonical or not.
By solving the above problems learners will be able to get clear idea about the conditions of canonical transformation.

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Conclusion:

The session can be concluded with a little GAME (Fastest Hand raising when you solve a cross-word puzzle)

Suggested Activity:

The following crossword puzzle will be placed in front of students to answer with a game Fastest Hand raising.

Prize: The winner with maximum answers gets a chocolate.

			1			2				5		
								4				
		3										
	6											
7												

Clues:

Sl.No.	Across:
3	In case of Canonical transformation……….equation of motion is preserved.
6	If (q,p) ⟶ (Q,P) is canonical then the passion bracket [Q,P] is equal to……….
7	If $pdq - pdQ$ is exact then the transformation (q,p) ⟶ (Q,P) is………….

Sl. No.	Down
1	In lagrengian mechanics transformation of the co-ordinates q ⟶ Q do not affect the form of………………..equations of motion.
2	In case of canonical transformation the form of……………may be changed.
4	A point transformation q ⟶ Q one modifies the…………co-ordinates.
5	The point transformation not…………be canonical.

Answer to the Crossword Puzzle:

			L 1			H 2				N 5		
			A			A				E		
			G			M				C		
			R			I		L 4		E		
		H 3	A	M	I	L	T	O	N	S		
			N			T		C		S		
			G			O		A		A		
	O 6	N	E			N		T		R		
			S			I		I		I		
						A		O		L		
						N		N		Y		
C 7	A	N	O	N	I	C	A	L				

Summary

In this session, we learnt:

Definition of Canonical Transformation.
Idea of Generating Function.
Condition for a Transformation to be Canonical.
Canonical Transformation in terms of Poisson Bracket.

Assignment

1. Check whether the following transformation is canonical or not

$$Q = \sqrt{2q}e^{a}cosp$$
$$P = \sqrt{2q}e^{a}sinp$$

2. For what values of m and n the following transformation is canonical

$$Q = q^{m}cosnp$$
$$P = q^{m}sinnp$$

References

Classical Mechanics – H. Goldstein.
Classical Mechanics –John R. Taylor
Classical Mechanics – Gupta, Kumar & Sharma